

Calculus mock exams

AP Calculus AB and BC + international challenges

Original student papers, worked solutions, and marking guides.

September 2026 edition | AP structure effective May 2027.

Paper	Student pages	Solution pages
AP Calculus AB	3–31	89–107
AP Calculus BC	32–60	108–126
Singapore-inspired	61–67	127–130
China-inspired	68–74	131–134
Japan-inspired	75–81	135–138
European approaches	82–88	139–142

How to use this collection

Start with a student paper and keep its solution pages closed. Time each part separately, then mark your work and return to the relevant video lesson. The PDF bookmarks jump to each paper and solution guide.

Page ranges above refer to the combined PDF page count. The footer numbers within each paper restart at 1. For classroom testing, use the separate student PDFs available on the website.

The AP mocks each contain 42 MCQs and 6 FRQs. Each international challenge contains 3 structured problems and 30 marks. The international papers are calculus enrichment, not full national examinations.

Format, scoring and references

AP timing and practice scoring

MCQ A: 29 questions in 62 minutes, no calculator. MCQ B: 13 questions in 38 minutes, graphing calculator required. FRQ A: 2 questions in 30 minutes, graphing calculator required. FRQ B: 4 questions in 60 minutes, no calculator.

Weighted practice score = $50 \times (\text{correct MCQs} / 42) + 50 \times (\text{FRQ points} / 54)$. This is a score out of 100 for revision, not a predicted AP 1–5 result. The real AP exam uses Bluebook; this collection provides printable practice.

International scope

Singapore: applied modeling, optimization and parametric calculus. China: parameters and derivative-based proof. Japan: integral recurrences and geometric reasoning. Europe: selected A-level and French-style themes involving models, sequences and proof. The 60-minute timings and 30-mark rubrics are designed for these practice sets; they are not national examination specifications.

Official reference links

[AP Calculus AB exam structure](#)

[AP Calculus BC exam structure](#)

[Singapore SEAB: 2027 syllabuses](#)

[China NEEA: Gaokao resources](#)

[University of Tokyo: past entrance papers](#)

[Cambridge International: A-level Mathematics 9709](#)

[France Eduscol: baccalaureat past-paper archive](#)

Reference review: September 17, 2026. All questions and practice rubrics are original. No content is reproduced from Barron's, The Princeton Review, or the linked exam boards. Independent material, without endorsement by those organizations. AP is a College Board trademark.

RIGOROUS CALCULUS PRACTICE

AP Calculus AB

Full-length mock | 2027 structure

Foundations through differential equations

Section	Questions / time	Calculator
MCQ Part A	29 / 62 minutes	Not permitted
MCQ Part B	13 / 38 minutes	Graphing required
FRQ Part A	2 / 30 minutes	Graphing required
FRQ Part B	4 / 60 minutes	Not permitted

Total working time: **190 minutes**. MCQ and FRQ each carry 50% of this mock's weighted practice score.

Before you begin

Write your name and date below. Keep the separate solutions file closed until you finish. Show equations, reasoning, units and required justifications. Use radian mode. Unless an exact answer is requested, round final numerical answers to three decimal places and retain unrounded values during calculations.

Choose exactly one answer for each MCQ. Use the answer sheet at the back of Section I. Time the four parts separately and keep the calculator closed in the two no-calculator parts. These printable pages do not simulate the Bluebook interface.

Name: _____ Date: _____

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Edition: September 17, 2026. AP structure effective May 2027. ChenMathAcademy.org

Section I | Part A**No calculator**

29 questions / 62 minutes

1. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(3x) - 3x}{x^3}$.

(A) $-\frac{3}{2}$

(B) 0

(C) $\frac{9}{2}$

(D) $-\frac{9}{2}$

2. For $x \neq 0$, let $f(x) = (e^{2x} - 1)/x$, and define $f(0) = k$. Which pair $(k, f'(0))$ makes f differentiable at 0?

(A) (2, 2)

(B) (1, 2)

(C) (0, 2)

(D) (2, 4)

3. A differentiable one-to-one function satisfies $f(2) = 9$ and $f'(2) = 5$. If $h(x) = f^{-1}(x^2)$, find $h'(3)$.

(A) $\frac{3}{5}$

(B) $\frac{1}{5}$

(C) $\frac{6}{5}$

(D) 30

Section I | Part A**No calculator**

29 questions / 62 minutes

4. The curve $x^2 + xy + y^2 = 7$ passes through $(1, 2)$. What is d^2y/dx^2 there?

(A) $-\frac{42}{125}$

(B) $\frac{42}{125}$

(C) $-\frac{4}{5}$

(D) $-\frac{2}{5}$

5. Let $f(x) = e^{\sin x}$. Which is the tangent line at $x = \pi$?

(A) $y = 1 + (x - \pi)$

(B) $y = e^\pi(x - \pi)$

(C) $y = -(x - \pi)$

(D) $y = 1 - (x - \pi)$

6. If $f(x) = x^x$ for $x > 0$, find $f'(2)$.

(A) $4(1 + \ln 2)$

(B) 8

(C) $2(1 + \ln 2)$

(D) $4 \ln 2$

Section I | Part A**No calculator**

29 questions / 62 minutes

7. Suppose $g(1) = 2$, $g'(1) = -3$, $f(2) = 5$, and $f'(2) = 4$. For $H(x) = f(g(x))/g(x)$, find $H'(1)$.

- (A) $-\frac{39}{4}$
- (B) $-\frac{9}{4}$
- (C) -6
- (D) $\frac{9}{4}$

8. A continuous function f has $f(3) = 7$. Evaluate $\lim_{h \rightarrow 0} \frac{1}{h} \int_3^{3+2h} f(t) dt$.

- (A) 7
- (B) $\frac{7}{2}$
- (C) 0
- (D) 14

9. For $F(x) = \int_1^{x^2} \ln(1+t^2) dt$, find $F'(1)$.

- (A) $\ln 2$
- (B) $2 \ln 2$
- (C) $4 \ln 2$
- (D) 2

Section I | Part A**No calculator**

29 questions / 62 minutes

10. Evaluate $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n} \left(1 + \frac{k}{n}\right)^2$.

(A) $\frac{1}{3}$

(B) 3

(C) $\frac{7}{3}$

(D) $\frac{8}{3}$

11. For $f(x) = |x|$ on $[-1, 1]$, why can the Mean Value Theorem not be applied?

(A) $f(-1) \neq f(1)$.

(B) The average rate of change is undefined.

(C) f is not differentiable at 0.(D) f is not continuous at 0.

12. For $f(x) = x^3 - 3x$ on $[-\sqrt{3}, \sqrt{3}]$, which values of c satisfy the conclusion of Rolle's theorem?

(A) $c = -1, 1$

(B) $c = 1$ only

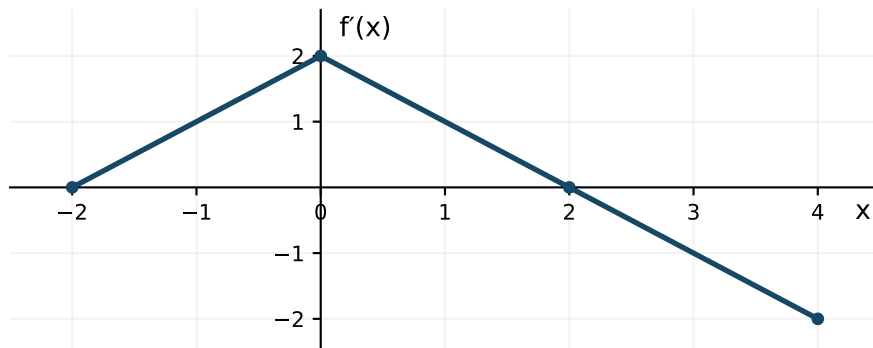
(C) $c = 0$

(D) $c = -\sqrt{3}, \sqrt{3}$

Section I | Part A**No calculator**

29 questions / 62 minutes

13. The graph shown is f' . Its line segments join $(-2, 0)$, $(0, 2)$, $(2, 0)$, and $(4, -2)$. If $f(0) = 1$, what is the absolute maximum value of f on $[-2, 4]$?



- (A) 3
- (B) 5
- (C) 2
- (D) 1

14. If $f'(x) = (x - 1)^2(x + 2)$, which statement is true?

- (A) f has a local maximum at 1.
- (B) f has local minima at both -2 and 1 .
- (C) f has a local maximum at -2 .
- (D) f has a local minimum at $x = -2$ and no extremum at $x = 1$.

Section I | Part A**No calculator**

29 questions / 62 minutes

15. Evaluate $\int_0^2 |x^2 - 1| dx$.

- (A) $\frac{4}{3}$
- (B) $\frac{2}{3}$
- (C) $\frac{8}{3}$
- (D) 2

16. What is the average value of $f(x) = \ln x/x$ on $[1, e]$?

- (A) $\frac{1}{2}$
- (B) $\frac{e-1}{2}$
- (C) $\frac{1}{e-1}$
- (D) $\frac{1}{2(e-1)}$

17. A twice-differentiable function has $f(0) = 2$, $f(4) = 10$, and $f''(x) > 0$ on $(0, 4)$. Which must hold?

- (A) $f'(0) > 2 > f'(4)$
- (B) $f'(x) = 2$ throughout
- (C) $f'(0) < 2 < f'(4)$
- (D) $f'(2) = 2$

Section I | Part A**No calculator**

29 questions / 62 minutes

18. A solid has base between $y = \sqrt{x}$ and $y = x$ on $0 \leq x \leq 1$. Cross sections perpendicular to the x -axis are squares. Find its volume.

- (A) $\frac{1}{6}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{30}$
- (D) $\frac{1}{15}$

19. The region between $y = x$ and $y = x^2$, $0 \leq x \leq 1$, is revolved about $y = 2$. Find the volume.

- (A) $\frac{2\pi}{15}$
- (B) $\frac{4\pi}{15}$
- (C) $\frac{\pi}{6}$
- (D) $\frac{8\pi}{15}$

20. The solution of $y' = xy^2$ with $y(0) = -1$ satisfies which equation?

- (A) $y = -1 + \frac{x^2}{2}$
- (B) $y = -e^{x^2/2}$
- (C) $y = \frac{2}{x^2-2}$
- (D) $y = -\frac{2}{x^2+2}$

Section I | Part A**No calculator**

29 questions / 62 minutes

21. A solution of $y' = y(3 - y)$ passes through $(0, 1)$. At that point, the solution is

- (A) decreasing and concave down.
- (B) decreasing and concave up.
- (C) increasing and concave down.
- (D) increasing and concave up.

22. Using the tangent line to $f(x) = \sqrt{x}$ at $x = 4$, the estimate of $\sqrt{4.2}$ is

- (A) 2.10, an overestimate.
- (B) 2.05, an overestimate.
- (C) 2.025, an underestimate.
- (D) 2.05, an underestimate.

23. Water fills a cone with radius always one-third of its water depth h . When $h = 6$ cm and $dh/dt = 4$ cm/s, find dV/dt .

- (A) $\frac{16\pi}{3}$ cm³/s
- (B) 48π cm³/s
- (C) 16π cm³/s
- (D) 144π cm³/s

Section I | Part A**No calculator**

29 questions / 62 minutes

24. A rectangle has area 30. Its horizontal edges cost 2 per unit length and its vertical edges cost 3 per unit length. Which horizontal length x minimizes the total cost?

- (A) $\frac{3\sqrt{5}}{2}$
- (B) $2\sqrt{5}$
- (C) $\sqrt{30}$
- (D) $3\sqrt{5}$

25. Evaluate $\int_0^{1/2} \frac{2}{1+4x^2} dx$.

- (A) $\frac{\pi}{2}$
- (B) $\frac{\pi}{4}$
- (C) $\frac{\pi}{8}$
- (D) $\ln 2$

26. Evaluate $\int_0^{\pi/2} \sin x \cos^3 x \, dx$.

- (A) 1
- (B) $\frac{3}{4}$
- (C) $\frac{1}{2}$
- (D) $\frac{1}{4}$

Section I | Part A**No calculator**

29 questions / 62 minutes

27. Evaluate $\lim_{x \rightarrow +\infty} (\sqrt{9x^2 + 2x} - 3x)$.

- (A) $\frac{1}{3}$
- (B) $+\infty$
- (C) $\frac{2}{3}$
- (D) 0

28. Define $f(0) = 0$ and $f(x) = x^2 \sin(1/x)$ for $x \neq 0$. Which statement is correct?

- (A) $f'(0) = 1$.
- (B) $f'(0) = 0$, but f' is not continuous at 0.
- (C) $f'(0)$ does not exist.
- (D) f' is continuous at 0.

29. For which real parameters k is $f(x) = x^3 - 3kx$ increasing on all of $\mathbb{R}$?

- (A) $k \geq 0$
- (B) $k < 0$ only
- (C) $k = 0$ only
- (D) $k \leq 0$

STOP. End of this part. Check your responses before moving to the next timed part.

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

30. Find the unique solution of $e^{-x} = x$ on $[0, 1]$.

- (A) 0.739
- (B) 0.567
- (C) 0.432
- (D) 0.693

31. The curves $y = e^{-x}$ and $y = x$ intersect at $x = a > 0$. Find the area enclosed by these curves and the y -axis.

- (A) 0.136
- (B) 0.472
- (C) 0.272
- (D) 0.728

32. A particle has velocity $v(t) = 2 \sin t - 0.4t$ for $0 \leq t \leq 5$. At what positive time does it change direction?

- (A) 3.096
- (B) 2.596
- (C) 2.096
- (D) 3.142

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

33. A particle has velocity $v(t) = t^3 - 4t + 1$ on $[0, 3]$. Find the total distance traveled.

- (A) 11.636
- (B) 5.250
- (C) 9.636
- (D) 4.818

34. A tank starts with 30 liters. Inflow is $5 + 2\sin(t^2)$ L/min and outflow is $4 + 0.5t$ L/min. How much water is in the tank at $t = 4$?

- (A) 35.494
- (B) 1.494
- (C) 51.494
- (D) 31.494

35. Find the average value of e^{-x^2} on $[0, 2]$.

- (A) 0.441
- (B) 0.221
- (C) 0.882
- (D) 0.632

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

36. For $f(x) = \ln(1 + x^2)$ on $[0, 3]$, find the larger number c satisfying the Mean Value Theorem conclusion.

- (A) 0.468
- (B) 2.138
- (C) 1.000
- (D) 3.000

37. The curve $y = \ln x$ intersects $y = 2 - x$ at $x = a$. Find the slope of the normal to $y = \ln x$ at that point.

- (A) 1.557
- (B) -0.642
- (C) 0.642
- (D) -1.557

38. A solution of $dy/dx = \sin x/y$ satisfies $y(0) = 2$. Find $y(1.8)$.

- (A) 6.454
- (B) 2.541
- (C) 1.270
- (D) 1.541

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

39. The region below $y = e^{-x^2}$ and above the x -axis on $[0, 2]$ is rotated about the x -axis. Find the volume.

- (A) 3.937
- (B) 0.627
- (C) 1.969
- (D) 2.771

40. The table gives a flow rate $r(t)$ in L/min. Estimate $\int_0^4 r(t)dt$ by the trapezoidal rule on the three listed subintervals.

t	0	1	3	4
$r(t)$	2	5	4	8

- (A) 18.500
- (B) 19.500
- (C) 17.000
- (D) 21.000

41. The curve $xe^y + y^2 = 5$ has a point with $x = 2$ and $y > 0$. Find dy/dx at that point.

- (A) -0.184
- (B) -2.716
- (C) -0.368
- (D) 0.368

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

42. A quantity starts at 0 and increases at rate $2 + \sin(t^2)$ units/min. When does it first reach 3 units?

- (A) 1.500
- (B) 1.735
- (C) 0.935
- (D) 1.235

STOP. End of this part. Check your responses before moving to the next timed part.

Multiple-choice answer sheet

Mark one response per question.

- | | | |
|---|---|---|
| 1. ☐ A ☐ B ☐ C ☐ D | 15. ☐ A ☐ B ☐ C ☐ D | 29. ☐ A ☐ B ☐ C ☐ D |
| 2. ☐ A ☐ B ☐ C ☐ D | 16. ☐ A ☐ B ☐ C ☐ D | 30. ☐ A ☐ B ☐ C ☐ D |
| 3. ☐ A ☐ B ☐ C ☐ D | 17. ☐ A ☐ B ☐ C ☐ D | 31. ☐ A ☐ B ☐ C ☐ D |
| 4. ☐ A ☐ B ☐ C ☐ D | 18. ☐ A ☐ B ☐ C ☐ D | 32. ☐ A ☐ B ☐ C ☐ D |
| 5. ☐ A ☐ B ☐ C ☐ D | 19. ☐ A ☐ B ☐ C ☐ D | 33. ☐ A ☐ B ☐ C ☐ D |
| 6. ☐ A ☐ B ☐ C ☐ D | 20. ☐ A ☐ B ☐ C ☐ D | 34. ☐ A ☐ B ☐ C ☐ D |
| 7. ☐ A ☐ B ☐ C ☐ D | 21. ☐ A ☐ B ☐ C ☐ D | 35. ☐ A ☐ B ☐ C ☐ D |
| 8. ☐ A ☐ B ☐ C ☐ D | 22. ☐ A ☐ B ☐ C ☐ D | 36. ☐ A ☐ B ☐ C ☐ D |
| 9. ☐ A ☐ B ☐ C ☐ D | 23. ☐ A ☐ B ☐ C ☐ D | 37. ☐ A ☐ B ☐ C ☐ D |
| 10. ☐ A ☐ B ☐ C ☐ D | 24. ☐ A ☐ B ☐ C ☐ D | 38. ☐ A ☐ B ☐ C ☐ D |
| 11. ☐ A ☐ B ☐ C ☐ D | 25. ☐ A ☐ B ☐ C ☐ D | 39. ☐ A ☐ B ☐ C ☐ D |
| 12. ☐ A ☐ B ☐ C ☐ D | 26. ☐ A ☐ B ☐ C ☐ D | 40. ☐ A ☐ B ☐ C ☐ D |
| 13. ☐ A ☐ B ☐ C ☐ D | 27. ☐ A ☐ B ☐ C ☐ D | 41. ☐ A ☐ B ☐ C ☐ D |
| 14. ☐ A ☐ B ☐ C ☐ D | 28. ☐ A ☐ B ☐ C ☐ D | 42. ☐ A ☐ B ☐ C ☐ D |

End of Section I.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 1: A storage tank

9 points

A tank contains 20 liters at $t = 0$. Water enters at $r(t) = 10e^{-0.2t}$ liters per minute and leaves at $s(t) = 2 + 0.4t$ liters per minute, for $0 \leq t \leq 8$. The tank has sufficient capacity.

(a) Find the total amount of water that enters during the first 8 minutes. Include units. [2]

(b) Find the amount of water in the tank at $t = 8$. [2]

Show sufficient work to support your answers.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 1: A storage tank

9 points

continued

Use the definitions and information on the preceding page.

(c) At $t = 4$, is the amount of water increasing or decreasing? Is its rate of change increasing or decreasing? Justify both answers. [2]

(d) Find the time at which the tank contains the greatest amount of water on $[0, 8]$, and find that amount. Justify that it is an absolute maximum. [3]

Show sufficient work to support your answers.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 2: Motion on a line

9 points

A particle moves on a line with velocity $v(t) = e^{-t/3} \sin t - 0.1$ meters per second for $0 \leq t \leq 6$. Its position at $t = 0$ is 2 meters. Use radian measure.

- (a) Find the position at $t = 6$ and the average velocity on $[0, 6]$. [2]

- (b) Find all times in $(0, 6)$ at which the particle changes direction. Explain using velocity signs. [2]

Show sufficient work to support your answers.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 2: Motion on a line**9 points**

continued

Use the definitions and information on the preceding page.

(c) Find the total distance traveled on $[0, 6]$.

[2]

(d) At $t = 4$, is the speed increasing or decreasing? Show the calculus reasoning.

[3]

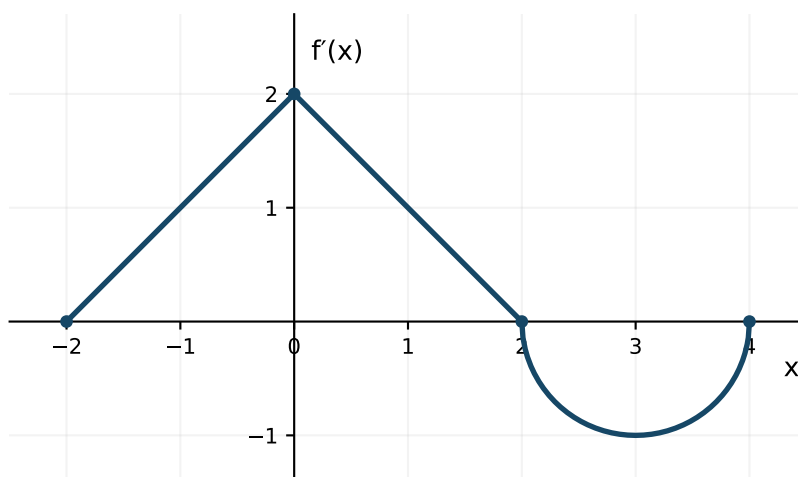
Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 3: Read a derivative graph**9 points**

The continuous graph of f' consists of line segments from $(-2, 0)$ to $(0, 2)$ to $(2, 0)$, followed by the lower semicircle of radius 1 centered at $(3, 0)$ from $x = 2$ to $x = 4$. Let $f(0) = 1$. The graph below is f' , not f .



(a) Find $f(-2)$ and $f(4)$ exactly.

[2]

(b) Find the absolute maximum and absolute minimum values of f on $[-2, 4]$. Justify.

[2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 3: Read a derivative graph**9 points**

continued

Use the definitions and information on the preceding page.

(c) Give every open interval on which f is concave up, and identify all inflection-point x -coordinates. Explain. [2]

(d) Let $h(x) = f(x^2)$. Find $h'(1)$ and $h''(1)$. [3]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 4: A separable differential equation

9 points

Let $y = f(x)$ solve $y' = (2x + 1)(y - 2)$ with $f(0) = 3$.

(a) Write the tangent line at $x = 0$, and use it to approximate $f(0.1)$. [2]

(b) Find $f''(0)$. Show the differentiation used. [2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 4: A separable differential equation**9 points**

continued

Use the definitions and information on the preceding page.

(c) Find the particular solution $f(x)$ by separation of variables.

[3]

(d) Is the tangent-line approximation in part (a) an overestimate or an underestimate? Justify on the whole interval $[0, 0.1]$, not just at 0.

[2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 5: One region, three solids

9 points

Let R be the region bounded by $y = 2x$ and $y = x^2$.

(a) Find the area of R . [2]

(b) A solid has base R and square cross sections perpendicular to the x -axis. Find its volume. [2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 5: One region, three solids

9 points

continued

Use the definitions and information on the preceding page.

(c) Write, but do not evaluate, a washer integral for the volume obtained by rotating R about $y = 4$. Identify the two radii. [2]

(d) Write and evaluate a washer integral for the volume obtained by rotating R about the y -axis. Use integration with respect to y . [3]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 6: Function values and theorems**9 points**

The function f is twice continuously differentiable. Selected values are shown, and $\int_0^3 f(x)dx = 6$.

x	0	1	2	3
$f(x)$	1	2	0	4
$f'(x)$	3	−1	2	5

(a) Let $H(x) = \int_1^{x^2} f(t)dt$. Find $H'(1)$ and $H''(1)$. [2]

(b) Explain why there is a $c \in (2, 3)$ with $f'(c) = 4$. Name the theorem and check its hypotheses. [2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 6: Function values and theorems

9 points

continued

Use the definitions and information on the preceding page.

(c) A local inverse g of f is chosen with $g(0) = 2$. Find $g'(0)$, and explain why this local inverse exists. [2]

(d) Evaluate $\int_0^3 [2f(x) + f'(x)]dx$. Show how the given integral and endpoint values enter the calculation. [3]

Show sufficient work to support your answers.

RIGOROUS CALCULUS PRACTICE

AP Calculus BC

Full-length mock | 2027 structure

AB concepts plus advanced integration, polar, parametric and series

Section	Questions / time	Calculator
MCQ Part A	29 / 62 minutes	Not permitted
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Total working time: **190 minutes**. MCQ and FRQ each carry 50% of this mock's weighted practice score.

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Choose exactly one answer for each MCQ. Use the answer sheet at the back of Section I. Time the four parts separately and keep the calculator closed in the two no-calculator parts. These printable pages do not simulate the Bluebook interface.

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Section I | Part A**No calculator**

29 questions / 62 minutes

1. Evaluate $\lim_{x \rightarrow 0} \frac{\ln(1 + 2x) - 2x}{x^2}$.

- (A) 0
- (B) -2
- (C) 2
- (D) -1

2. A one-to-one function has $f(1) = 3$, $f'(1) = 2$, and $f''(1) = -4$. For $g = f^{-1}$, find $g''(3)$.

- (A) $-\frac{1}{2}$
- (B) $\frac{1}{2}$
- (C) -2
- (D) 2

3. For $x^2 + y^3 = 9$, find y'' at $(1, 2)$.

- (A) $-\frac{1}{6}$
- (B) $-\frac{7}{18}$
- (C) $\frac{7}{36}$
- (D) $-\frac{7}{36}$

Section I | Part A**No calculator**

29 questions / 62 minutes

4. If $F(x) = \int_x^{2x} e^{-t^2} dt$, find $F'(0)$.

- (A) 1
- (B) 0
- (C) 2
- (D) -1

5. A differentiable function has continuous derivative, $f(0) = 1$, and $f(2) = 7$. What is the average value of f' on $[0, 2]$?

- (A) 6
- (B) $\frac{7}{2}$
- (C) 4
- (D) 3

6. If $f'(x) = (x + 1)(x - 2)^2$, where is f concave down?

- (A) $(-\infty, 0)$
- (B) $(0, 2)$
- (C) $(2, \infty)$
- (D) $(-1, 2)$

Section I | Part A**No calculator**

29 questions / 62 minutes

7. What are the absolute minimum and maximum values of $f(x) = x + 4/x$ on $[1, 4]$?

- (A) minimum 4, maximum 8
- (B) minimum 2, maximum 5
- (C) minimum 4, maximum 5
- (D) minimum 5, maximum 8

8. A particle has velocity $v(t) = t^2 - 6t + 5$ for $0 < t < 6$. On which intervals is its speed increasing?

- (A) $(0, 1) \cup (3, 5)$
- (B) $(3, 6)$
- (C) $(0, 3)$
- (D) $(1, 3) \cup (5, 6)$

9. If $\int_1^5 f(u)du = 8$, find $\int_1^3 f(2x - 1)dx$.

- (A) 4
- (B) 8
- (C) 16
- (D) 2

Section I | Part A**No calculator**

29 questions / 62 minutes

10. Find the area between $y = x$ and $y = x^3$ for $0 \leq x \leq 1$.

- (A) $\frac{1}{2}$
- (B) $\frac{1}{12}$
- (C) $\frac{3}{4}$
- (D) $\frac{1}{4}$

11. A solid has base the disk $x^2 + y^2 \leq 1$. Its cross sections perpendicular to the x -axis are equilateral triangles with bases in the disk. Find its volume.

- (A) $\frac{4\sqrt{3}}{3}$
- (B) $\sqrt{3}$
- (C) $\frac{2\sqrt{3}}{3}$
- (D) π

12. The region below $y = \sqrt{x}$, above $y = 0$, and between $x = 1$ and $x = 4$ is rotated about the x -axis. Find the volume.

- (A) 15π
- (B) 3π
- (C) 7π
- (D) $\frac{15\pi}{2}$

Section I | Part A**No calculator**

29 questions / 62 minutes

13. Solve $y' = 2x(y + 1)$ with $y(0) = 2$.

- (A) $y = 2e^{x^2}$
- (B) $y = e^{x^2} + 1$
- (C) $y = 3e^{x^2} - 1$
- (D) $y = 3e^{2x} - 1$

14. Find the average value of $\sec^2 x$ on $[0, \pi/4]$.

- (A) 1
- (B) $\frac{\pi}{4}$
- (C) $\frac{4}{\pi}$
- (D) $\frac{2}{\pi}$

15. For $x > 0$, what is the minimum value of $f(x) = x^2 \ln x$?

- (A) 0
- (B) $-\frac{1}{e}$
- (C) $-\frac{1}{2e}$
- (D) $-\frac{1}{e^2}$

Section I | Part A**No calculator**

29 questions / 62 minutes

16. For $x \neq 0$, suppose $-|x| \leq f(x) \leq |x|$. What must be true?

- (A) $f'(0) = 0$
- (B) $\lim_{x \rightarrow 0} f(x) = 0$
- (C) f is differentiable near 0.
- (D) $f(0) = 0$

17. An antiderivative of $1/[x(x+2)]$ is

- (A) $\ln|x| - \ln|x+2| + C$
- (B) $\frac{1}{2} \ln|x+2| + C$
- (C) $\frac{1}{2} \ln \left| \frac{x}{x+2} \right| + C$
- (D) $\ln|x(x+2)| + C$

18. Evaluate $\int_0^1 x \ln(1+x) dx$.

- (A) $\frac{1}{2}$
- (B) $\frac{1}{4}$
- (C) $\frac{\ln 2}{2}$
- (D) $\ln 2 - \frac{1}{2}$

Section I | Part A**No calculator**

29 questions / 62 minutes

19. Evaluate $\int_e^\infty \frac{dx}{x(\ln x)^2}$.

- (A) 0
- (B) e^{-1}
- (C) The integral diverges.
- (D) 1

20. A curve has $x = t^2 + 1$, $y = t^3 - 3t$. Find d^2y/dx^2 at $t = 1$.

- (A) 6
- (B) 0
- (C) $\frac{3}{2}$
- (D) 3

21. Find the area enclosed by the polar curve $r = 2 \cos \theta$, traced once for $-\pi/2 \leq \theta \leq \pi/2$.

- (A) 2π
- (B) π
- (C) 4π
- (D) $\frac{\pi}{2}$

Section I | Part A**No calculator**

29 questions / 62 minutes

22. A particle has velocity $\mathbf{v}(t) = \langle e^t, 2t \rangle$. Find the rate of change of its speed at $t = 0$.

- (A) 2
- (B) 0
- (C) $\sqrt{5}$
- (D) 1

23. Which statement describes $\sum_{n=1}^{\infty} \frac{n!}{3^n}$?

- (A) It diverges because its terms do not approach zero.
- (B) It converges conditionally.
- (C) It converges by the ratio test.
- (D) It is a convergent geometric series.

24. Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$.

- (A) $[-1, 5]$
- (B) $[-1, 5)$
- (C) $(-1, 5)$
- (D) $(-1, 5]$

Section I | Part A**No calculator**

29 questions / 62 minutes

25. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ is

- (A) divergent because $1/\sqrt{n}$ diverges.
- (B) conditionally convergent.
- (C) divergent because the terms fail to approach zero.
- (D) absolutely convergent.

26. Find the coefficient of x^5 in the Maclaurin series for $x^2 \sin(3x)$.

- (A) $-\frac{27}{120}$
- (B) $\frac{9}{2}$
- (C) 3
- (D) $-\frac{9}{2}$

27. What is the radius of convergence of the Maclaurin series for $1/(4 + x^2)$?

- (A) 2
- (B) ∞
- (C) $\frac{1}{2}$
- (D) 4

Section I | Part A**No calculator**

29 questions / 62 minutes

28. Use $x - x^3/3$ to approximate $\arctan x$ at $x = 1/2$. What alternating-series bound follows for the absolute error?

- (A) $\frac{1}{448}$
- (B) $\frac{1}{32}$
- (C) $\frac{1}{24}$
- (D) $\frac{1}{160}$

29. Evaluate $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$.

- (A) 1
- (B) $\frac{1}{2}$
- (C) $\frac{3}{4}$
- (D) $\frac{3}{2}$

STOP. End of this part. Check your responses before moving to the next timed part.

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

30. Find the positive solution of $\ln x + x^2 = 3$.

- (A) 1.842
- (B) 1.732
- (C) 1.592
- (D) 1.342

31. The region below $y = \ln(1 + x^2)$ on $0 \leq x \leq 1$ is rotated about the x -axis. Find the volume.

- (A) 0.364
- (B) 0.729
- (C) 0.693
- (D) 0.116

32. Find the area enclosed by $y = e^{-x}$, $y = x^2$, and the y -axis in the first quadrant.

- (A) 0.539
- (B) 0.611
- (C) 0.389
- (D) 0.195

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

33. Find the absolute maximum value of $f(x) = xe^{-x^2} + 0.1x$ on $[0, 3]$.

- (A) 0.300
- (B) 0.768
- (C) 0.235
- (D) 0.503

34. For $f(x) = \sin(x^2)$, find the average value of f' on $[1, 3]$.

- (A) -0.215
- (B) 0.215
- (C) 0.107
- (D) -0.429

35. Let $f(x) = x^3 + x + 1$ and $g = f^{-1}$. Find $g'(5)$.

- (A) 0.448
- (B) 6.703
- (C) 0.149
- (D) 1.379

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

36. The table gives $f(x)$. Estimate $\int_0^5 [f(x)]^2 dx$ with trapezoids on the three listed subintervals.

x	0	1	2	5
$f(x)$	1	2	3	2

- (A) 28.500
- (B) 24.500
- (C) 32.500
- (D) 12.000

37. A particle has velocity $v(t) = \sin t - 0.2$ on $[0, 2\pi]$. Find total distance traveled.

- (A) 1.757
- (B) 4.000
- (C) 4.080
- (D) 1.257

38. Use Euler's method with step $h = 0.2$ for $y' = x + y$, $y(0) = 1$, to estimate $y(0.6)$.

- (A) 1.728
- (B) 1.480
- (C) 1.856
- (D) 1.822

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

39. Find the arc length of $x = t^2$, $y = \ln(1 + t)$ for $0 \leq t \leq 2$.

- (A) 4.148
- (B) 4.335
- (C) 2.168
- (D) 5.335

40. In the first quadrant, find the area inside both $r = 1 + \cos \theta$ and $r = 2 \sin \theta$.

- (A) 1.571
- (B) 0.505
- (C) 1.010
- (D) 2.020

41. A population satisfies $P' = 0.3P(1 - P/500)$ and $P(0) = 50$. When does it first reach 250?

- (A) 5.365
- (B) 7.324
- (C) 3.662
- (D) 2.197

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

42. Use the degree-3 Maclaurin polynomial for e^{2x} at $x = 0.4$. What Lagrange error bound follows using the maximum fourth derivative on $[0, 0.4]$?

- (A) 0.608
- (B) 0.017
- (C) 0.038
- (D) 0.002

STOP. End of this part. Check your responses before moving to the next timed part.

Multiple-choice answer sheet

Mark one response per question.

- | | | |
|---|---|---|
| 1. ☐ A ☐ B ☐ C ☐ D | 15. ☐ A ☐ B ☐ C ☐ D | 29. ☐ A ☐ B ☐ C ☐ D |
| 2. ☐ A ☐ B ☐ C ☐ D | 16. ☐ A ☐ B ☐ C ☐ D | 30. ☐ A ☐ B ☐ C ☐ D |
| 3. ☐ A ☐ B ☐ C ☐ D | 17. ☐ A ☐ B ☐ C ☐ D | 31. ☐ A ☐ B ☐ C ☐ D |
| 4. ☐ A ☐ B ☐ C ☐ D | 18. ☐ A ☐ B ☐ C ☐ D | 32. ☐ A ☐ B ☐ C ☐ D |
| 5. ☐ A ☐ B ☐ C ☐ D | 19. ☐ A ☐ B ☐ C ☐ D | 33. ☐ A ☐ B ☐ C ☐ D |
| 6. ☐ A ☐ B ☐ C ☐ D | 20. ☐ A ☐ B ☐ C ☐ D | 34. ☐ A ☐ B ☐ C ☐ D |
| 7. ☐ A ☐ B ☐ C ☐ D | 21. ☐ A ☐ B ☐ C ☐ D | 35. ☐ A ☐ B ☐ C ☐ D |
| 8. ☐ A ☐ B ☐ C ☐ D | 22. ☐ A ☐ B ☐ C ☐ D | 36. ☐ A ☐ B ☐ C ☐ D |
| 9. ☐ A ☐ B ☐ C ☐ D | 23. ☐ A ☐ B ☐ C ☐ D | 37. ☐ A ☐ B ☐ C ☐ D |
| 10. ☐ A ☐ B ☐ C ☐ D | 24. ☐ A ☐ B ☐ C ☐ D | 38. ☐ A ☐ B ☐ C ☐ D |
| 11. ☐ A ☐ B ☐ C ☐ D | 25. ☐ A ☐ B ☐ C ☐ D | 39. ☐ A ☐ B ☐ C ☐ D |
| 12. ☐ A ☐ B ☐ C ☐ D | 26. ☐ A ☐ B ☐ C ☐ D | 40. ☐ A ☐ B ☐ C ☐ D |
| 13. ☐ A ☐ B ☐ C ☐ D | 27. ☐ A ☐ B ☐ C ☐ D | 41. ☐ A ☐ B ☐ C ☐ D |
| 14. ☐ A ☐ B ☐ C ☐ D | 28. ☐ A ☐ B ☐ C ☐ D | 42. ☐ A ☐ B ☐ C ☐ D |

End of Section I.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 1: Rates measured at uneven times

9 points

A reservoir contains 40 cubic meters at $t = 0$. Its inflow $r(t)$ is twice continuously differentiable, with values in the table. Outflow is $s(t) = 2 + 0.25t$ cubic meters per hour. Let $B(t)$ be the water volume.

t (hours)	0	2	5	8
$r(t)$ (m ³ /hour)	4	7	5	1

- (a) Use trapezoids on the three listed subintervals to estimate the total inflow on $[0, 8]$. [2]

- (b) Use your estimate to approximate $B(8)$. [2]

Show sufficient work to support your answers.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 1: Rates measured at uneven times

9 points

continued

Use the definitions and information on the preceding page.

(c) Use the secant slope of r on $[2, 8]$ to estimate $r'(5)$. Then estimate $B''(5)$ and give its units. [2]

(d) Prove that $B'(c) = 0$ for at least one $c \in (5, 8)$. Do the data establish a unique absolute maximum of B on $[0, 8]$? Explain. [3]

Show sufficient work to support your answers.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 2: A planar trajectory

9 points

A particle starts at $(1, -2)$ at $t = 0$ and has velocity components $x'(t) = 1 + \cos(t^2)$ and $y'(t) = \sin t + 0.3$ for $0 \leq t \leq 3$. Position is measured in meters and time in seconds.

(a) Find the particle's position at $t = 3$. [2]

(b) Find the speed and acceleration vector at $t = 2$. [2]

Show sufficient work to support your answers.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 2: A planar trajectory

9 points

continued

Use the definitions and information on the preceding page.

(c) Find the total distance traveled on $[0, 3]$. [2]

(d) Find all times in $(0, 3)$ when the trajectory has a vertical tangent. Justify that the tangent is vertical at each listed time. [3]

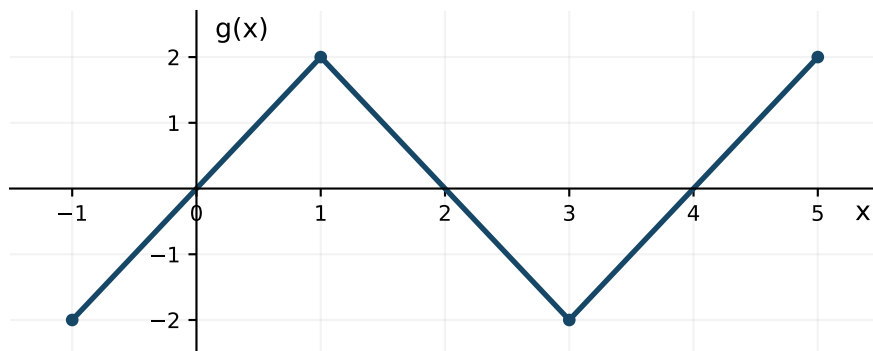
Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 3: An accumulation function**9 points**

A continuous function g is defined on $[-1, 5]$ by the line segments joining $(-1, -2)$, $(1, 2)$, $(3, -2)$, and $(5, 2)$. Let $A(x) = \int_{-1}^x g(t) dt$.



(a) Find $A(1)$ and $A(4)$. Use signed areas.

[2]

(b) Find every local maximum and local minimum of A in $(-1, 5)$. Give each coordinate $(x, A(x))$ and justify.

[2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 3: An accumulation function

9 points

continued

Use the definitions and information on the preceding page.

(c) On what intervals is A concave down? Where does its concavity change? [2]

(d) Let $J(x) = \int_0^{A(x)} e^{u^2} du$. Find $J'(2)$ and $J''(2)$. [3]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 4: A differential equation and its interval

9 points

Let $y = f(x)$ solve $y' = xy^2$ with $f(0) = 2$.

(a) Use Euler's method with two steps of size 0.25 to estimate $f(0.5)$. Show both updates. [2]

(b) Find the particular solution by separation of variables. [2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 4: A differential equation and its interval

9 points

continued

Use the definitions and information on the preceding page.

(c) Find the largest open interval containing 0 on which this solution is finite and differentiable. [2]

(d) Determine whether the Euler estimate in part (a) is below or above the exact value. Justify using the concavity of the solution on $[0, 0.5]$. [3]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 5: Polar geometry

9 points

Consider the polar curve $r = 2 + \cos \theta$ for $0 \leq \theta \leq 2\pi$. It is traced once and r is positive throughout.

(a) Find dy/dx at $\theta = \pi/2$. Show the parametric derivatives. [2]

(b) Find the total area enclosed by the curve. [2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 5: Polar geometry**9 points**

continued

Use the definitions and information on the preceding page.

(c) Find the area inside this curve and outside the circle $r = 2$.

[3]

(d) Write, but do not evaluate, an integral for the total arc length of the polar curve.

[2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 6: Series, endpoints, and an error bound

9 points

Let $F(x) = \int_0^x \ln(1+t^2)dt$. You may use $\ln(1+u) = \sum_{n=1}^{\infty} (-1)^{n+1} u^n/n$ for $-1 < u \leq 1$.

(a) Write the first three nonzero terms and the general term of the Maclaurin series for F . [2]

(b) Find the interval of convergence of this series. Justify what happens at both endpoints. [2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 6: Series, endpoints, and an error bound

9 points

continued

Use the definitions and information on the preceding page.

- (c) Use the first two nonzero terms to approximate $F(1/2)$. Give an exact fraction for the approximation.
[2]

- (d) Give an alternating-series upper bound for the absolute error in part (c). Is that approximation above or below the true value? Justify.
[3]

Show sufficient work to support your answers.

RIGOROUS CALCULUS PRACTICE

Singapore-inspired

Calculus challenge mock

Modeling, optimization and parametric reasoning

60 minutes | 3 structured questions | 30 marks

A graphing calculator may be used for numerical work. Exact answers and proofs are still required where requested.

This is a calculus-focused enrichment paper inspired by selected exam approaches. It is not a full national examination or a claim of syllabus equivalence. Europe has many different examination systems.

Before you begin

Write your name and date below. Keep the separate solutions file closed until you finish. Show equations, reasoning, units and required justifications. Use radian mode. Unless an exact answer is requested, round final numerical answers to three decimal places and retain unrounded values during calculations.

Name: _____ Date: _____

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Edition: September 17, 2026. ChenMathAcademy.org

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 1: Design an open box

10 marks

A $30\text{ cm} \times 20\text{ cm}$ rectangle of card is used to make an open box. Squares of side $x\text{ cm}$ are removed from its four corners, and the sides are folded up. Ignore the card thickness.

- (a) Give the physical domain of x and show that the volume is $V(x) = 4x^3 - 100x^2 + 600x$. [2]

- (b) Find the value of x giving the greatest volume. Give an exact value and a decimal approximation. [3]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 1: Design an open box

10 marks

continued

Use the definitions and information on the preceding page.

(c) Prove that the stationary point you selected is the absolute maximum on the physical domain. Find the maximum volume. [3]

(d) A designer chooses $x = 4$ cm. Calculate the percentage loss in volume compared with the optimum, and explain why the loss is relatively small near a stationary maximum. [2]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 2: Cooling and model interpretation

10 marks

A drink cools in a room at a constant 22°C . Its temperature satisfies $T' = -k(T - 22)$, where $k > 0$ and t is in minutes. Initially $T(0) = 90$ and $T(10) = 56$.

(a) Solve the differential equation and determine k exactly. [3]

(b) Find the predicted temperature at $t = 25$. [2]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 2: Cooling and model interpretation

10 marks

continued

Use the definitions and information on the preceding page.

(c) Find the average temperature during the first 20 minutes. Give an exact expression and a decimal value. [3]

(d) Find when the model first predicts $T = 30$. State one physical assumption that matters when using this prediction. [2]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 3: A parametric region

10 marks

A curve is given by $x = t^2$ and $y = t^3 - t$ for $t \geq 0$. Let R be the finite region between the curve and the x -axis for $0 \leq t \leq 1$.

(a) Find the horizontal and vertical tangent points for $t \geq 0$. Use derivative conditions to justify them. [3]

(b) Find the exact area of R . [2]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 3: A parametric region

10 marks

continued

Use the definitions and information on the preceding page.

(c) Find the exact volume obtained by rotating R about the x -axis. [3]

(d) For $t > 0$, show that $d^2y/dx^2 > 0$ and interpret the result geometrically. [2]

Show sufficient work to support your answers.

RIGOROUS CALCULUS PRACTICE

China-inspired

Calculus challenge mock

Parameter cases, root counting and inequality proofs

60 minutes | 3 structured questions | 30 marks

No calculator. Give exact answers and complete mathematical justifications.

This is a calculus-focused enrichment paper inspired by selected exam approaches. It is not a full national examination or a claim of syllabus equivalence. Europe has many different examination systems.

Before you begin

Write your name and date below. Keep the separate solutions file closed until you finish. Show equations, reasoning, units and required justifications. Use radian mode. Unless an exact answer is requested, round final numerical answers to three decimal places and retain unrounded values during calculations.

Name: _____ Date: _____

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Structured challenge

No calculator | 60 minutes for all three questions

Question 1: A logarithm with a parameter

10 marks

For $a > 0$, define $f_a(x) = \ln x - a(x - 1)$ on $x > 0$. Give exact answers and justify all parameter cases.

(a) Find the intervals of increase and decrease and the maximum value of f_a . [3]

(b) Prove that $f_a(x) \leq 0$ for every $x > 0$ if and only if $a = 1$. [3]

Show sufficient work to support your answers.

Structured challenge

No calculator | 60 minutes for all three questions

Question 1: A logarithm with a parameter

10 marks

continued

Use the definitions and information on the preceding page.

(c) For fixed $a > 0$, determine the number of positive solutions of $\ln x - ax = b$ for every real b . [2]

(d) When $a = 1$, show that $f_1(x) = 0$ has exactly one solution, and explain why this zero is not a crossing of the x -axis. [2]

Show sufficient work to support your answers.

Structured challenge

No calculator | 60 minutes for all three questions

Question 2: An absolute maximum for every parameter

10 marks

Let $g_a(x) = x^3 - 3ax$ on $[-1, 1]$, where a is any real number. Define $M(a) = \max_{-1 \leq x \leq 1} g_a(x)$.

(a) Determine $M(a)$ when $a \leq 0$ and when $a \geq 1$. Include where it is attained. [2]

(b) For $0 < a < 1$, find all interior stationary points and classify them. List every candidate value for the absolute maximum. [3]

Show sufficient work to support your answers.

Structured challenge

No calculator | 60 minutes for all three questions

Question 2: An absolute maximum for every parameter

10 marks

continued

Use the definitions and information on the preceding page.

(c) Prove that the local maximum value is at least $g_a(-1)$, and find when it equals $g_a(1)$. [3]

(d) Give a complete piecewise formula for $M(a)$ for all real a , identifying the maximizing point or points in every case. [2]

Show sufficient work to support your answers.

Structured challenge

No calculator | 60 minutes for all three questions

Question 3: The sharp constant in an inequality

10 marks

For $x > 0$, define $q(x) = [x - \ln(1 + x)]/x^2$. The word *sharp* means the largest constant for which an inequality holds throughout the stated interval.

(a) Prove that $\ln(1 + x) < x$ for every $x > 0$. [2]

(b) Show that $q'(x) < 0$ for $x > 0$. You may analyze the derivative of $H(x) = x^2/(1 + x) - 2x + 2\ln(1 + x)$. [3]

Show sufficient work to support your answers.

Structured challenge

No calculator | 60 minutes for all three questions

Question 3: The sharp constant in an inequality

10 marks

continued

Use the definitions and information on the preceding page.

(c) Find the sharp constant C such that $\ln(1+x) \leq x - Cx^2$ for all $0 < x \leq 1$. Prove both validity and sharpness. [3]

(d) Find $\lim_{x \rightarrow 0^+} q(x)$ and hence state strict lower and upper bounds for $q(x)$ on $0 < x < 1$. [2]

Show sufficient work to support your answers.

RIGOROUS CALCULUS PRACTICE

Japan-inspired

Calculus challenge mock

Integral recurrences, symmetry and geometric proofs

60 minutes | 3 structured questions | 30 marks

No calculator. Give exact answers and complete mathematical justifications.

This is a calculus-focused enrichment paper inspired by selected exam approaches. It is not a full national examination or a claim of syllabus equivalence. Europe has many different examination systems.

Before you begin

Write your name and date below. Keep the separate solutions file closed until you finish. Show equations, reasoning, units and required justifications. Use radian mode. Unless an exact answer is requested, round final numerical answers to three decimal places and retain unrounded values during calculations.

Name: _____ Date: _____

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Edition: September 17, 2026. ChenMathAcademy.org

Structured challenge

No calculator | 60 minutes for all three questions

Question 1: An integral recurrence and a limit

10 marks

For each integer $n \geq 0$, let $I_n = \int_0^1 x^n/(1+x) dx$. All answers must be justified without numerical integration.

(a) Find I_0 and prove $I_n + I_{n+1} = 1/(n+1)$. [3]

(b) Use the recurrence to find I_3 exactly. [2]

Show sufficient work to support your answers.

Structured challenge

No calculator | 60 minutes for all three questions

Question 1: An integral recurrence and a limit

10 marks

continued

Use the definitions and information on the preceding page.

(c) Prove $0 \leq I_n - 1/[2(n+1)] \leq 1/[2(n+1)(n+2)]$. [3]

(d) Deduce $\lim_{n \rightarrow \infty} (n+1)I_n$ and $\lim_{n \rightarrow \infty} I_{n+1}/I_n$. [2]

Show sufficient work to support your answers.

Structured challenge

No calculator | 60 minutes for all three questions

Question 2: Oscillation and a volume invariant

10 marks

For a positive integer n , let R_n be the region between $y = |\sin(nx)|$ and the x -axis on $0 \leq x \leq \pi$. Let V_n be its volume of revolution about the y -axis. You may use the cylindrical-shell formula $V = 2\pi \int x h(x) dx$, where h is the nonnegative shell height.

(a) Find the area of R_n and prove that it does not depend on n . [2]

(b) Show that $|\sin(n(\pi - x))| = |\sin(nx)|$. [2]

Show sufficient work to support your answers.

Structured challenge

No calculator | 60 minutes for all three questions

Question 2: Oscillation and a volume invariant

10 marks

continued

Use the definitions and information on the preceding page.

(c) Let $J_n = \int_0^\pi x |\sin(nx)| dx$. Use the substitution $u = \pi - x$ to find J_n exactly. [3]

(d) Find V_n . Also find the volume obtained by rotating R_n about the x -axis, and explain whether either volume depends on n . [3]

Show sufficient work to support your answers.

Structured challenge

No calculator | 60 minutes for all three questions

Question 3: Two tangents to a parabola

10 marks

Let $a < b$. Tangents to $y = x^2$ at $x = a$ and $x = b$, together with the parabolic arc between those points, enclose a finite region. Its lower boundary is the upper of the two tangent lines.

(a) Find the tangent equations and their intersection point. [2]

(b) Prove algebraically that each tangent lies on or below the parabola for every real x . [2]

Show sufficient work to support your answers.

Structured challenge

No calculator | 60 minutes for all three questions

Question 3: Two tangents to a parabola

10 marks

continued

Use the definitions and information on the preceding page.

(c) Prove that the area of the enclosed region is $(b - a)^3/12$. Your integral must account for the change in lower boundary. [4]

(d) The tangent intersection has x -coordinate 1 and the enclosed area is 18. Determine a , b , and the intersection's y -coordinate. [2]

Show sufficient work to support your answers.

RIGOROUS CALCULUS PRACTICE

European approaches

Calculus challenge mock

Selected A-level and French-style themes: models, sequences and proof

60 minutes | 3 structured questions | 30 marks

A graphing calculator may be used for numerical work. Exact answers and proofs are still required where requested.

This is a calculus-focused enrichment paper inspired by selected exam approaches. It is not a full national examination or a claim of syllabus equivalence. Europe has many different examination systems.

Before you begin

Write your name and date below. Keep the separate solutions file closed until you finish. Show equations, reasoning, units and required justifications. Use radian mode. Unless an exact answer is requested, round final numerical answers to three decimal places and retain unrounded values during calculations.

Name: _____ Date: _____

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Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 1: Exponential growth with a carrying capacity

10 marks

A population is modeled by $P' = 0.2P(1 - P/800)$, with $P(0) = 100$. Time is measured in years. This is a continuous population model; intermediate values need not be integers.

(a) Find the equilibrium solutions. For the given initial value, state the sign of P' and the long-term population predicted by the model. [2]

(b) Use separation of variables to show that $P(t) = 800/(1 + 7e^{-0.2t})$. [3]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 1: Exponential growth with a carrying capacity

10 marks

continued

Use the definitions and information on the preceding page.

(c) Find when the growth rate is greatest and calculate that greatest rate. [3]

(d) Find the first time the population reaches 600. Explain one reason a real population might depart from the long-term prediction. [2]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 2: Newton iteration and proof of convergence

10 marks

Let $u_0 = 2$ and $u_{n+1} = \frac{1}{2}(u_n + 2/u_n)$ for $n \geq 0$. The sequence is intended to approximate $\sqrt{2}$.

(a) Derive this iteration by applying Newton's method to $f(x) = x^2 - 2$, and calculate u_1 and u_2 exactly.
[2]

(b) Prove by induction that $u_n > \sqrt{2}$ for all n . You may derive an identity for $u_{n+1} - \sqrt{2}$. [3]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 2: Newton iteration and proof of convergence

10 marks

continued

Use the definitions and information on the preceding page.

(c) Prove that the sequence decreases and converges to $\sqrt{2}$. [3]

(d) Show that $0 < u_{n+1} - \sqrt{2} \leq (u_n - \sqrt{2})^2 / (2\sqrt{2})$. Explain what this says about the error near the limit. [2]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 3: A response curve and its parameters

10 marks

A nonnegative response is modeled by $C(t) = Ate^{-kt}$ for $t \geq 0$, where $A, k > 0$. Time is in hours. Measurements show that the peak occurs at $t = 2$ and the peak value is 12.

(a) Find the peak time and peak value in terms of A and k , with a derivative sign argument. [2]

(b) Determine A and k from the measurements. [2]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 3: A response curve and its parameters

10 marks

continued

Use the definitions and information on the preceding page.

(c) Find the total response $\int_0^\infty C(t)dt$ and the fraction of this total that occurs in the first 2 hours. Give exact answers. [4]

(d) If A is held fixed and k doubles, explain how the peak time, peak value, and total response change. [2]

Show sufficient work to support your answers.

RIGOROUS CALCULUS PRACTICE

AP Calculus AB

Worked solutions and marking guide

Foundations through differential equations

Section	Questions / time	Calculator
MCQ Part A	29 / 62 minutes	Not permitted
MCQ Part B	13 / 38 minutes	Graphing required
FRQ Part A	2 / 30 minutes	Graphing required
FRQ Part B	4 / 60 minutes	Not permitted

Total working time: **190 minutes**. MCQ and FRQ each carry 50% of this mock's weighted practice score.

How to mark fairly

Award credit for mathematically equivalent methods. The point allocations are Academy practice rubrics, not official exam-board rubrics. Give method credit for sound work even if a later arithmetic error occurs; avoid penalizing the same error twice. Do not award a reasoning point for an unsupported correct number.

Practice score: Let m be the number of correct MCQs out of 42, and r the total FRQ points out of 54. Compute $50(m/42) + 50(r/54)$ out of 100. There is no penalty for an incorrect MCQ. This score is not an AP 1–5 prediction; no official grade boundaries are supplied.

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Quick answer key

Question	Answer	Question	Answer	Question	Answer
1	D	15	D	29	D
2	A	16	D	30	B
3	C	17	C	31	C
4	A	18	C	32	B
5	D	19	D	33	C
6	A	20	D	34	D
7	B	21	D	35	A
8	D	22	B	36	B
9	B	23	C	37	D
10	C	24	D	38	B
11	C	25	B	39	C
12	A	26	D	40	A
13	A	27	A	41	C
14	D	28	B	42	D

Diagnose before you repeat

For each missed question, record the topic, the incorrect step, and the idea that fixes it. Rework the question without reading the explanation. Then return to the relevant Academy video and its lesson worksheet.

Multiple-choice worked solutions

1. Answer D

Limits

Correct choice: $-\frac{9}{2}$

The quotient has form $0/0$. Three applications of L'Hospital give $\lim[-27 \cos(3x)/6] = -9/2$. Each intermediate quotient has form $0/0$.

2. Answer A

Continuity and differentiability

Correct choice: $(2, 2)$

Continuity requires $k = 2$. Then $f'(0) = \lim_{h \rightarrow 0} (e^{2h} - 1 - 2h)/h^2 = 2$ by two applications of L'Hospital.

3. Answer C

Inverse functions

Correct choice: $\frac{6}{5}$

Chain rule: $h'(x) = 2x/f'(f^{-1}(x^2))$. At $x = 3$, the inverse input is 2, so $h'(3) = 6/5$.

4. Answer A

Implicit differentiation

Correct choice: $-\frac{42}{125}$

First $2x + y + (x + 2y)y' = 0$, giving $y' = -4/5$. Differentiate again: $2 + 2y' + 2(y')^2 + (x + 2y)y'' = 0$. Substitute to obtain $y'' = -42/125$.

Multiple-choice worked solutions

5. Answer D

Chain rule

Correct choice: $y = 1 - (x - \pi)$

$f(\pi) = 1$ and $f'(x) = e^{\sin x} \cos x$, so the slope at π is -1 .

6. Answer A

Logarithmic differentiation

Correct choice: $4(1 + \ln 2)$

Logarithmic differentiation gives $\ln f = x \ln x$, so $f'/f = 1 + \ln x$ and $f'(2) = 4(1 + \ln 2)$.

7. Answer B

Chain and quotient rules

Correct choice: $-\frac{9}{4}$

$H' = g'[gf'(g) - f(g)]/g^2$. Substitute: $(-3)(2 \cdot 4 - 5)/4 = -9/4$.

8. Answer D

FTC and limits

Correct choice: 14

The numerator is zero at $h = 0$ and its derivative is $2f(3 + 2h)$ by FTC and chain rule. The limit is $2f(3) = 14$.

Multiple-choice worked solutions

9. Answer B

FTC

Correct choice: $2 \ln 2$

$F'(x) = 2x \ln(1 + x^4)$, hence $F'(1) = 2 \ln 2$.

10. Answer C

Riemann sums

Correct choice: $\frac{7}{3}$

This is a right-endpoint Riemann sum for $\int_1^2 x^2 dx = [x^3/3]_1^2 = 7/3$.

11. Answer C

MVT hypotheses

Correct choice: f is not differentiable at 0.

The function is continuous on the closed interval, but has a corner at 0. In fact the secant slope is 0, whereas every existing derivative on the interior is -1 or 1 .

12. Answer A

Rolle theorem

Correct choice: $c = -1, 1$

The polynomial is continuous and differentiable, and its endpoint values are both 0. Solve $f'(c) = 3c^2 - 3 = 0$: $c = \pm 1$, both interior.

Multiple-choice worked solutions

13. Answer A

Derivative graph

Correct choice: 3

$f' > 0$ on $(-2, 2)$ and $f' < 0$ on $(2, 4)$, so the maximum occurs at 2. The area under f' from 0 to 2 is a triangle of area 2, so $f(2) = 1 + 2 = 3$.

14. Answer D

Derivative sign analysis

Correct choice: f has a local minimum at $x = -2$ and no extremum at $x = 1$.

The squared factor does not change sign. The derivative changes from negative to positive at -2 , but is positive on both sides of 1.

15. Answer D

Absolute value integrals

Correct choice: 2

Split at 1: $\int_0^1 (1 - x^2) dx = 2/3$ and $\int_1^2 (x^2 - 1) dx = 4/3$. Their sum is 2.

16. Answer D

Average value

Correct choice: $\frac{1}{2(e-1)}$

Average value is $(e-1)^{-1} \int_1^e (\ln x)/x dx$. An antiderivative is $(\ln x)^2/2$, giving $1/[2(e-1)]$.

Multiple-choice worked solutions

17. Answer C

MVT and concavity

Correct choice: $f'(0) < 2 < f'(4)$

The secant slope is 2. MVT gives an interior c with $f'(c) = 2$, and $f'' > 0$ makes f' strictly increasing, giving the strict endpoint inequalities.

18. Answer C

Cross-section volumes

Correct choice: $\frac{1}{30}$

Area of a square slice is $(\sqrt{x} - x)^2 = x - 2x^{3/2} + x^2$. Integrate from 0 to 1: $1/2 - 4/5 + 1/3 = 1/30$.

19. Answer D

Washer method

Correct choice: $\frac{8\pi}{15}$

Outer radius $2 - x^2$ and inner radius $2 - x$ give $V = \pi \int_0^1 [(2 - x^2)^2 - (2 - x)^2] dx = \pi(2 - 5/3 + 1/5) = 8\pi/15$.

20. Answer D

Separation of variables

Correct choice: $y = -\frac{2}{x^2+2}$

Separate: $y^{-2} dy = x dx$. Then $-1/y = x^2/2 + C$ and $C = 1$, giving $y = -2/(x^2 + 2)$.

Multiple-choice worked solutions

21. Answer D

Differential-equation analysis

Correct choice: increasing and concave up.

At the point, $y' = 2 > 0$. Differentiating yields $y'' = (3 - 2y)y' = 2 > 0$.

22. Answer B

Linearization

Correct choice: 2.05, an overestimate.

$L(x) = 2 + (x - 4)/4$, giving 2.05. Since $f''(x) < 0$, the graph lies below this tangent line.

23. Answer C

Related rates

Correct choice: $16\pi \text{ cm}^3/\text{s}$

$V = \pi h^3/27$. Thus $dV/dt = (\pi h^2/9)(dh/dt)$; at the instant this equals 16π .

24. Answer D

Optimization

Correct choice: $3\sqrt{5}$

With height $30/x$, cost $C = 4x + 180/x$. Set $C' = 4 - 180/x^2 = 0$, giving $x = \sqrt{45} = 3\sqrt{5}$. Since $C'' = 360/x^3 > 0$, this is the minimum.

Multiple-choice worked solutions

25. Answer B

Inverse trig integrals

Correct choice: $\frac{\pi}{4}$

Use $u = 2x$. The integral becomes $\int_0^1 (1 + u^2)^{-1} du = \arctan 1 - \arctan 0 = \pi/4$.

26. Answer D

Substitution

Correct choice: $\frac{1}{4}$

With $u = \cos x$, $du = -\sin x \, dx$ and bounds 1 to 0. The value is $\int_0^1 u^3 du = 1/4$.

27. Answer A

Limits at infinity

Correct choice: $\frac{1}{3}$

Rationalize: $2x/(\sqrt{9x^2 + 2x} + 3x) = 2/(\sqrt{9 + 2/x} + 3) \rightarrow 1/3$.

28. Answer B

Differentiability versus continuity

Correct choice: $f'(0) = 0$, but f' is not continuous at 0.

The difference quotient is $h \sin(1/h) \rightarrow 0$. For $x \neq 0$, $f'(x) = 2x \sin(1/x) - \cos(1/x)$; its oscillating cosine term prevents a limit at 0.

Multiple-choice worked solutions

29. Answer D

Parameter analysis

Correct choice: $k \leq 0$

$f'(x) = 3(x^2 - k)$. For $k < 0$ it is positive everywhere; for $k = 0$, x^3 remains strictly increasing. If $k > 0$, the derivative is negative near 0.

30. Answer B

Numerical zeros

Correct choice: 0.567

Use a numerical zero of $e^{-x} - x$. Its derivative $-e^{-x} - 1 < 0$ and endpoint signs differ, so the root is unique. Numerically, the answer is 0.567143, which rounds to 0.567.

31. Answer C

Area between curves

Correct choice: 0.272

Solve $e^{-a} = a$, giving $a \approx 0.567143290$. Area $= \int_0^a (e^{-x} - x) dx = 1 - a - a^2/2$. Numerically, the answer is 0.272031, which rounds to 0.272.

32. Answer B

Motion

Correct choice: 2.596

Find the nonzero root of $2 \sin t - 0.4t = 0$. Velocity changes from positive to negative there; $t = 0$ is only an endpoint. Numerically, the answer is 2.595739, which rounds to 2.596.

Multiple-choice worked solutions

33. Answer C

Total distance

Correct choice: 9.636

Velocity zeros in the interval are 0.254102 and 1.860806. Integrate $|v|$, splitting at both roots, and add positive distances. Numerically, the answer is 9.636005, which rounds to 9.636.

34. Answer D

Accumulation model

Correct choice: 31.494

Use initial amount plus accumulated net rate: $30 + \int_0^4 [1 + 2 \sin(t^2) - 0.5t] dt$. Numerically, the answer is 31.494268, which rounds to 31.494.

35. Answer A

Average value

Correct choice: 0.441

The average value is $\frac{1}{2} \int_0^2 e^{-x^2} dx$. Use numerical integration and divide by the interval length. Numerically, the answer is 0.441041, which rounds to 0.441.

36. Answer B

MVT numerical solution

Correct choice: 2.138

The secant slope is $\ln(10)/3$. Solve $2c/(1+c^2) = \ln(10)/3$; both positive roots lie in $(0, 3)$, and select the larger. Numerically, the answer is 2.138051, which rounds to 2.138.

Multiple-choice worked solutions

37. Answer D

Normal line

Correct choice: -1.557

Solve $\ln a = 2 - a$. The tangent slope is $1/a$, so the normal slope is its negative reciprocal, $-a$. Numerically, the answer is -1.557146 , which rounds to -1.557 .

38. Answer B

Differential equations

Correct choice: 2.541

Separate and integrate: $y^2/2 = -\cos x + C$. The initial value gives $y^2 = 6 - 2\cos x$; choose the positive branch. Numerically, the answer is 2.540552 , which rounds to 2.541 .

39. Answer C

Numerical volume

Correct choice: 1.969

Use disks with squared radius: $V = \pi \int_0^2 (e^{-x^2})^2 dx = \pi \int_0^2 e^{-2x^2} dx$. Numerically, the answer is 1.968577 , which rounds to 1.969 .

40. Answer A

Tabular integration

Correct choice: 18.500

Use the actual widths: $1(2 + 5)/2 + 2(5 + 4)/2 + 1(4 + 8)/2 = 18.5$ liters. Numerically, the answer is 18.500000 , which rounds to 18.500 .

Multiple-choice worked solutions

41. Answer C

Implicit slope

Correct choice: -0.368

First solve $2e^y + y^2 = 5$, obtaining $y \approx 0.784825523$. Implicit differentiation gives $y' = -e^y/(xe^y + 2y)$. Numerically, the answer is -0.368179 , which rounds to -0.368 .

42. Answer D

Accumulation inverse problem

Correct choice: 1.235

Solve $\int_0^t [2 + \sin(u^2)] du = 3$ numerically. The integrand is at least 1, so the accumulated amount is strictly increasing and the time is unique. Numerically, the answer is 1.234677 , which rounds to 1.235 .

Free-response solution 1

A storage tank

Accumulation, rates, global extrema

9 points

(a) [2] The total inflow is $\int_0^8 10e^{-0.2t} dt = 50(1 - e^{-1.6}) \approx 39.905$ liters.

Marking: 1 point for the inflow integral; 1 for the value with units.

(b) [2] $W(8) = 20 + \int_0^8 [10e^{-0.2t} - 2 - 0.4t] dt \approx 31.105$ liters. Initial amount must be included.

Marking: 1 point for initial amount plus net accumulation; 1 for the value.

(c) [2] $W'(4) = 10e^{-0.8} - 3.6 \approx 0.893 > 0$, so the amount increases. $W''(4) = -2e^{-0.8} - 0.4 < 0$, so its rate of change decreases.

Marking: 1 point for each correctly justified conclusion.

(d) [3] $W'(t) = 0$ gives $t = c \approx 4.722$ minutes. Because $W''(t) = -2e^{-0.2t} - 0.4 < 0$, the net rate decreases strictly, is positive before c , and negative after c . Therefore the unique absolute maximum is $W(c) \approx 36.651$ liters.

Marking: 1 point for the critical time; 1 for the sign-change/global justification; 1 for the maximum amount.

Equivalent sound methods earn the same credit.

Free-response solution 2

Motion on a line

Motion, numerical integration, speed

9 points

(a) [2] $x(6) = 2 + \int_0^6 v(t)dt \approx 2.194$ meters. Average velocity is $[x(6) - 2]/6 \approx 0.032$ m/s.

Marking: 1 point for position with its integral; 1 for average velocity.

(b) [2] The two roots are $t \approx 0.104$ and 2.878 . The velocity signs are negative, positive, negative on the three resulting intervals. Each root is a change of direction.

Marking: 1 point for both roots; 1 for the signs and direction-change conclusion.

(c) [2] $D = -\int_0^{0.103702982} v(t)dt + \int_{0.103702982}^{2.877577344} v(t)dt - \int_{2.877577344}^6 v(t)dt \approx 1.646$ meters. Equivalently integrate $|v|$.

Marking: 1 point for an absolute-value or correctly split integral; 1 for the distance.

(d) [3] $a(t) = e^{-t/3}[\cos t - \frac{1}{3}\sin t]$. At $t = 4$, $v(4) \approx -0.299 < 0$ and $a(4) \approx -0.106 < 0$. Velocity and acceleration have the same sign, so speed increases.

Marking: 1 point for the acceleration formula; 1 for both signs; 1 for the speed conclusion.

Equivalent sound methods earn the same credit.

Free-response solution 3

Read a derivative graph

Graphical analysis and chain rule

9 points

(a) [2] $f(-2) = 1 - 2 = -1$. From 0 to 2 the signed area is 2; from 2 to 4 it is $-\pi/2$. Thus $f(4) = 3 - \pi/2$.

Marking: 1 point for each exact value with signed areas.

(b) [2] $f' > 0$ on $(-2, 2)$ and $f' < 0$ on $(2, 4)$, so the maximum is $f(2) = 3$. Compare the endpoints: $f(-2) = -1 < f(4) = 3 - \pi/2$, giving minimum -1 .

Marking: 1 point for the maximum with signs; 1 for the endpoint comparison and minimum.

(c) [2] f' increases on $(-2, 0)$ and $(3, 4)$, so f is concave up there. The derivative changes from increasing to decreasing at 0, and from decreasing to increasing at 3, so inflection points occur at $x = 0, 3$. There is no concavity change at 2.

Marking: 1 point for both intervals; 1 for both inflection coordinates with reasoning.

(d) [3] $h'(x) = 2xf'(x^2)$ and $h''(x) = 2f'(x^2) + 4x^2f''(x^2)$. The line segment gives $f'(1) = 1$ and $f''(1) = -1$, so $h'(1) = 2$ and $h''(1) = -2$.

Marking: 1 point for the first derivative and value; 1 for the second-derivative formula; 1 for its value.

Equivalent sound methods earn the same credit.

Free-response solution 4

A separable differential equation

Differential equations and tangent error

9 points

(a) [2] $f'(0) = 1$, so $L(x) = 3 + x$ and $f(0.1) \approx 3.1$.

Marking: 1 point for the line; 1 for the approximation.

(b) [2] $y'' = 2(y - 2) + (2x + 1)y'$. At $(0, 3)$, this gives $f''(0) = 2 + 1 = 3$.

Marking: 1 point for the derivative formula; 1 for the evaluation.

(c) [3] $dy/(y-2) = (2x+1)dx$ gives $\ln|y-2| = x^2+x+C$. Since $y(0)-2 = 1$, the solution is $f(x) = 2+e^{x^2+x}$.

Marking: 1 point for separation; 1 for integration; 1 for the particular solution.

(d) [2] $f''(x) = [2 + (2x + 1)^2]e^{x^2+x} > 0$ throughout $[0, 0.1]$. The curve is strictly concave up and lies above its tangent, so 3.1 is an underestimate.

Marking: 1 point for positivity on the interval; 1 for the justified underestimate.

Equivalent sound methods earn the same credit.

Free-response solution 5

One region, three solids

Area, cross sections, axis changes

9 points

(a) [2] Intersections satisfy $x^2 = 2x$, giving $x = 0, 2$. Area $= \int_0^2 (2x - x^2) dx = [x^2 - x^3/3]_0^2 = 4/3$.

Marking: 1 point for the integral and bounds; 1 for the area.

(b) [2] The side is $2x - x^2$, so $V = \int_0^2 (2x - x^2)^2 dx = [4x^3/3 - x^4 + x^5/5]_0^2 = 16/15$.

Marking: 1 point for squared cross-sectional area; 1 for the volume.

(c) [2] Outer radius is $4 - x^2$ and inner radius is $4 - 2x$. Thus $V = \pi \int_0^2 [(4 - x^2)^2 - (4 - 2x)^2] dx$.

Marking: 1 point for both radii; 1 for the correct integral.

(d) [3] At height y , the left and right boundaries are $x = y/2$ and $x = \sqrt{y}$, for $0 \leq y \leq 4$. Therefore $V = \pi \int_0^4 [y - y^2/4] dy = \pi [y^2/2 - y^3/12]_0^4 = 8\pi/3$.

Marking: 1 point for boundaries and limits; 1 for the washer integral; 1 for the volume.

Equivalent sound methods earn the same credit.

Free-response solution 6

Function values and theorems

Table, FTC, inverse and MVT

9 points

(a) [2] $H'(x) = 2xf(x^2)$, so $H'(1) = 4$. Also $H''(x) = 2f(x^2) + 4x^2f'(x^2)$, giving $H''(1) = 4 - 4 = 0$.

Marking: 1 point for each value with its derivative formula.

(b) [2] f is continuous on $[2, 3]$ and differentiable on $(2, 3)$. The Mean Value Theorem gives $f'(c) = [f(3) - f(2)]/(3 - 2) = 4$.

Marking: 1 point for the hypotheses; 1 for the theorem and secant calculation.

(c) [2] Because $f'(2) = 2 \neq 0$ and f' is continuous, f is strictly increasing near 2 and has a differentiable local inverse. Then $g'(0) = 1/f'(2) = 1/2$.

Marking: 1 point for local invertibility reasoning; 1 for the derivative.

(d) [3] By linearity and FTC, $\int_0^3 [2f + f'] dx = 2 \int_0^3 f dx + f(3) - f(0) = 2(6) + 4 - 1 = 15$.

Marking: 1 point for linearity; 1 for FTC endpoint difference; 1 for the value.

Equivalent sound methods earn the same credit.

RIGOROUS CALCULUS PRACTICE

AP Calculus BC

Worked solutions and marking guide

AB concepts plus advanced integration, polar, parametric and series

Section	Questions / time	Calculator
MCQ Part A	29 / 62 minutes	Not permitted
MCQ Part B	13 / 38 minutes	Graphing required
FRQ Part A	2 / 30 minutes	Graphing required
FRQ Part B	4 / 60 minutes	Not permitted

Total working time: **190 minutes**. MCQ and FRQ each carry 50% of this mock's weighted practice score.

How to mark fairly

Award credit for mathematically equivalent methods. The point allocations are Academy practice rubrics, not official exam-board rubrics. Give method credit for sound work even if a later arithmetic error occurs; avoid penalizing the same error twice. Do not award a reasoning point for an unsupported correct number.

Practice score: Let m be the number of correct MCQs out of 42, and r the total FRQ points out of 54. Compute $50(m/42) + 50(r/54)$ out of 100. There is no penalty for an incorrect MCQ. This score is not an AP 1–5 prediction; no official grade boundaries are supplied.

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Edition: September 17, 2026. AP structure effective May 2027. ChenMathAcademy.org

Quick answer key

Question	Answer	Question	Answer	Question	Answer
1	B	15	C	29	C
2	B	16	B	30	C
3	D	17	C	31	A
4	A	18	B	32	C
5	D	19	D	33	D
6	B	20	C	34	A
7	C	21	B	35	C
8	D	22	D	36	A
9	A	23	A	37	C
10	D	24	B	38	C
11	A	25	B	39	B
12	D	26	D	40	C
13	C	27	A	41	B
14	C	28	D	42	C

Diagnose before you repeat

For each missed question, record the topic, the incorrect step, and the idea that fixes it. Rework the question without reading the explanation. Then return to the relevant Academy video and its lesson worksheet.

Multiple-choice worked solutions

1. Answer B

Limits

Correct choice: -2

One L'Hospital step gives $[-4x/(1+2x)]/(2x) = -2/(1+2x)$ for $x \neq 0$, so the limit is -2 .

2. Answer B

Inverse derivatives

Correct choice: $\frac{1}{2}$

Differentiate $g'(y) = 1/f'(g(y))$: $g''(y) = -f''(g(y))/[f'(g(y))]^3$. Thus $g''(3) = 4/8 = 1/2$.

3. Answer D

Implicit derivatives

Correct choice: $-\frac{7}{36}$

$2x + 3y^2y' = 0$, so $y' = -1/6$. Then $2 + 6y(y')^2 + 3y^2y'' = 0$; substituting gives $y'' = -7/36$.

4. Answer A

FTC variable limits

Correct choice: 1

$F'(x) = 2e^{-4x^2} - e^{-x^2}$ by the chain rule at both endpoints. At 0 it is $2 - 1 = 1$.

Multiple-choice worked solutions

5. Answer D

Net change

Correct choice: 3

$$\text{Average derivative} = \frac{1}{2} \int_0^2 f'(x) dx = [f(2) - f(0)]/2 = 3.$$

6. Answer B

Concavity

Correct choice: $(0, 2)$

$$f'' = (x - 2)^2 + 2(x + 1)(x - 2) = 3x(x - 2), \text{ negative exactly on } (0, 2).$$

7. Answer C

Absolute extrema

Correct choice: minimum 4, maximum 5

$$f' = 1 - 4/x^2 \text{ vanishes at } 2. \text{ Compare } f(1) = 5, f(2) = 4, \text{ and } f(4) = 5.$$

8. Answer D

Motion and speed

Correct choice: $(1, 3) \cup (5, 6)$

Speed increases when velocity and acceleration have the same sign. $v = (t - 1)(t - 5)$ and $a = 2(t - 3)$ give both negative on $(1, 3)$ and both positive on $(5, 6)$.

Multiple-choice worked solutions

9. Answer A

Substitution

Correct choice: 4

Let $u = 2x - 1$, so $du = 2dx$ and bounds become 1, 5. The value is $\frac{1}{2} \int_1^5 f(u) du = 4$.

10. Answer D

Area

Correct choice: $\frac{1}{4}$

$x \geq x^3$ on this interval, so $\text{area} = \int_0^1 (x - x^3) dx = 1/2 - 1/4 = 1/4$.

11. Answer A

Cross sections

Correct choice: $\frac{4\sqrt{3}}{3}$

Triangle side $= 2\sqrt{1 - x^2}$, so $\text{area} = (\sqrt{3}/4) \cdot 4(1 - x^2)$. Integrate from -1 to 1 to obtain $4\sqrt{3}/3$.

12. Answer D

Volumes

Correct choice: $\frac{15\pi}{2}$

Disk area $\pi(\sqrt{x})^2 = \pi x$. Thus $V = \pi[x^2/2]_1^4 = 15\pi/2$.

Multiple-choice worked solutions

13. Answer C

Separable equations

Correct choice: $y = 3e^{x^2} - 1$

Separate: $dy/(y+1) = 2x dx$. Then $y+1 = Ce^{x^2}$; the initial condition gives $C = 3$.

14. Answer C

Average value

Correct choice: $\frac{4}{\pi}$

Average = $(4/\pi)[\tan x]_0^{\pi/4} = 4/\pi$.

15. Answer C

Optimization

Correct choice: $-\frac{1}{2e}$

$f'(x) = x(2\ln x + 1)$ changes from negative to positive at $x = e^{-1/2}$. At that point $f = -1/(2e)$.

16. Answer B

Squeeze theorem

Correct choice: $\lim_{x \rightarrow 0} f(x) = 0$

Both bounding functions tend to zero, so the squeeze theorem determines the limit. The assumptions do not define $f(0)$ or impose differentiability.

Multiple-choice worked solutions

17. Answer C

Partial fractions

Correct choice: $\frac{1}{2} \ln \left| \frac{x}{x+2} \right| + C$

$1/[x(x+2)] = \frac{1}{2}(1/x - 1/(x+2))$. Integrate both terms on any interval avoiding 0, -2.

18. Answer B

Integration by parts

Correct choice: $\frac{1}{4}$

Integrate by parts with $u = \ln(1+x)$ and $dv = x dx$. Use $x^2/(1+x) = x - 1 + 1/(1+x)$. The result is $\frac{1}{2} \ln 2 - \frac{1}{2}(\ln 2 - 1/2) = 1/4$.

19. Answer D

Improper integrals

Correct choice: 1

With $u = \ln x$, the integral becomes $\int_1^\infty u^{-2} du = [-u^{-1}]_1^\infty = 1$.

20. Answer C

Parametric derivatives

Correct choice: $\frac{3}{2}$

$dy/dx = \frac{3}{2}(t - 1/t)$. Differentiate in t and divide by $dx/dt = 2t$: $d^2y/dx^2 = \frac{3(1+t^{-2})}{4t}$, yielding $3/2$.

Multiple-choice worked solutions

21. Answer B

Polar area

Correct choice: π

$A = \frac{1}{2} \int_{-\pi/2}^{\pi/2} 4 \cos^2 \theta \, d\theta = \pi$. The curve is a circle of radius 1.

22. Answer D

Vector motion

Correct choice: 1

The speed derivative is $(\mathbf{v} \cdot \mathbf{a})/|\mathbf{v}|$. At zero, $\mathbf{v} = \langle 1, 0 \rangle$ and $\mathbf{a} = \langle 1, 2 \rangle$, so it equals 1.

23. Answer A

Series divergence

Correct choice: It diverges because its terms do not approach zero.

The consecutive-term ratio is $(n+1)/3$, eventually greater than 1 and unbounded. Terms eventually grow and cannot tend to zero.

24. Answer B

Power-series endpoints

Correct choice: $[-1, 5)$

The ratio test gives $|x-2| < 3$. At $x = -1$ the alternating harmonic series converges; at $x = 5$ the harmonic series diverges.

Multiple-choice worked solutions

25. Answer B

Conditional convergence

Correct choice: conditionally convergent.

The magnitudes decrease to zero, so the alternating-series test applies. The absolute series is a divergent p -series with $p = 1/2$.

26. Answer D

Taylor coefficients

Correct choice: $-\frac{9}{2}$

$$x^2 \sin(3x) = x^2(3x - (3x)^3/3! + \cdots) = 3x^3 - (9/2)x^5 + \cdots.$$

27. Answer A

Geometric power series

Correct choice: 2

$$1/(4 + x^2) = \frac{1}{4} \sum_{n=0}^{\infty} (-x^2/4)^n. \text{ Require } x^2/4 < 1, \text{ or } |x| < 2.$$

28. Answer D

Taylor error

Correct choice: $\frac{1}{160}$

The first omitted term is $x^5/5$. At $x = 1/2$ it is $1/(32 \cdot 5) = 1/160$.

Multiple-choice worked solutions

29. Answer C

Telescoping

Correct choice: $\frac{3}{4}$

$1/[n(n+2)] = \frac{1}{2}(1/n - 1/(n+2))$. Partial sums reduce to $\frac{1}{2}(1 + 1/2 - 1/(N+1) - 1/(N+2)) \rightarrow 3/4$.

30. Answer C

Numerical root

Correct choice: 1.592

The function $\ln x + x^2 - 3$ is strictly increasing for $x > 0$, and a numerical root gives its unique zero. Numerically, the answer is 1.592143, which rounds to 1.592.

31. Answer A

Numerical volume

Correct choice: 0.364

Use disks: $V = \pi \int_0^1 [\ln(1+x^2)]^2 dx$. Numerically, the answer is 0.364390, which rounds to 0.364.

32. Answer C

Area

Correct choice: 0.389

The positive intersection is $a \approx 0.703467422$. Integrate upper minus lower: $\int_0^a (e^{-x} - x^2) dx$. Numerically, the answer is 0.389093, which rounds to 0.389.

Multiple-choice worked solutions

33. Answer D

Absolute extrema

Correct choice: 0.503

Solve $f'(x) = e^{-x^2}(1 - 2x^2) + 0.1 = 0$. Critical numbers are approximately 0.768257 and 2.083658. Compare function values there and at both endpoints. Numerically, the answer is 0.502598, which rounds to 0.503.

34. Answer A

Average derivative

Correct choice: -0.215

By FTC, the average is $[f(3) - f(1)]/(3 - 1) = (\sin 9 - \sin 1)/2$. Use radian mode. Numerically, the answer is -0.214676 , which rounds to -0.215 .

35. Answer C

Inverse derivative

Correct choice: 0.149

Solve $f(a) = 5$, giving $a \approx 1.378796700$. The inverse derivative is $g'(5) = 1/f'(a) = 1/(3a^2 + 1)$. Numerically, the answer is 0.149182, which rounds to 0.149.

36. Answer A

Tabular integration

Correct choice: 28.500

Square the function values first. Then $1(1 + 4)/2 + 1(4 + 9)/2 + 3(9 + 4)/2 = 28.5$. Numerically, the answer is 28.500000, which rounds to 28.500.

Multiple-choice worked solutions

37. Answer C

Total distance

Correct choice: 4.080

Break at $\arcsin(0.2)$ and $\pi - \arcsin(0.2)$. Integrate the absolute value of velocity across the three sign intervals. Numerically, the answer is 4.080270, which rounds to 4.080.

38. Answer C

Euler method

Correct choice: 1.856

Use successive left-end slopes: $y_1 = 1 + 0.2(1) = 1.2$; $y_2 = 1.2 + 0.2(0.2 + 1.2) = 1.48$; $y_3 = 1.48 + 0.2(0.4 + 1.48) = 1.856$. Numerically, the answer is 1.856000, which rounds to 1.856.

39. Answer B

Parametric arc length

Correct choice: 4.335

Length $= \int_0^2 \sqrt{(2t)^2 + [1/(1+t)]^2} dt$. Integrate numerically. Numerically, the answer is 4.335285, which rounds to 4.335.

40. Answer C

Polar intersection area

Correct choice: 1.010

The crossing angle is $\alpha \approx 0.927295218$. The smaller radius is $2 \sin \theta$ before α and $1 + \cos \theta$ after it. Use $\frac{1}{2} \int_0^\alpha (2 \sin \theta)^2 d\theta + \frac{1}{2} \int_\alpha^{\pi/2} (1 + \cos \theta)^2 d\theta$. Numerically, the answer is 1.009921, which rounds to 1.010.

Multiple-choice worked solutions

41. Answer B

Logistic growth

Correct choice: 7.324

The solution is $P(t) = 500/(1 + 9e^{-0.3t})$. Setting $P = 250$ gives $9e^{-0.3t} = 1$, hence $t = \ln 9/0.3$. Numerically, the answer is 7.324082, which rounds to 7.324.

42. Answer C

Lagrange remainder

Correct choice: 0.038

$f^{(4)}(x) = 16e^{2x}$ has maximum $16e^{0.8}$. Thus $|R_3(0.4)| \leq 16e^{0.8}(0.4)^4/4! = e^{0.8}(0.8)^4/24$. Numerically, the answer is 0.037983, which rounds to 0.038.

Free-response solution 1

Rates measured at uneven times

Tables, rates, IVT, limits of inference

9 points

(a) [2] $\int_0^8 r(t)dt \approx 2(4+7)/2 + 3(7+5)/2 + 3(5+1)/2 = 38$ cubic meters.

Marking: 1 point for trapezoids with actual widths; 1 for the estimate.

(b) [2] $\int_0^8 s(t)dt = [2t + t^2/8]_0^8 = 24$. Thus $B(8) \approx 40 + 38 - 24 = 54$ cubic meters.

Marking: 1 point for total outflow; 1 for initial volume plus net change.

(c) [2] $r'(5) \approx [r(8) - r(2)]/(8 - 2) = -1$. Since $B'' = r' - s'$, $B''(5) \approx -1 - 0.25 = -1.25$ cubic meters per hour squared.

Marking: 1 point for the secant estimate; 1 for the second derivative with units.

(d) [3] $B' = r - s$ is continuous; $B'(5) = 5 - 3.25 = 1.75 > 0$ and $B'(8) = 1 - 4 = -3 < 0$. The Intermediate Value Theorem guarantees a zero between them. Values at four times do not control all intervening rate changes, so they cannot establish a unique absolute maximum.

Marking: 1 point for continuity; 1 for endpoint signs and IVT; 1 for the limitation.

Equivalent sound methods earn the same credit.

Free-response solution 2

A planar trajectory

Vector motion and parametric curves

9 points

(a) [2] $x(3) = 1 + \int_0^3 [1 + \cos(t^2)] dt \approx 4.703$ and $y(3) = -2 + \int_0^3 [\sin t + 0.3] dt \approx 0.890$.

Marking: 1 point for each coordinate with initial position and integral.

(b) [2] Speed is $\sqrt{(1 + \cos 4)^2 + (\sin 2 + 0.3)^2} \approx 1.258$ m/s. Acceleration is $\langle -2t \sin(t^2), \cos t \rangle$, so $\mathbf{a}(2) \approx \langle 3.027, -0.416 \rangle$ m/s².

Marking: 1 point for speed; 1 for the acceleration vector.

(c) [2] $D = \int_0^3 \sqrt{[1 + \cos(t^2)]^2 + [\sin t + 0.3]^2} dt \approx 5.082$ meters.

Marking: 1 point for the speed integral; 1 for the distance.

(d) [3] A vertical tangent requires $x' = 0$ and $y' \neq 0$. Here $t^2 = (2k + 1)\pi$; only $t = \sqrt{\pi} \approx 1.772$ lies in $(0, 3)$, since $\sqrt{3\pi} > 3$. At that time $y' = \sin \sqrt{\pi} + 0.3 > 0$.

Marking: 1 point for the condition; 1 for the unique time; 1 for nonzero vertical velocity.

Equivalent sound methods earn the same credit.

Free-response solution 3

An accumulation function

Accumulation, graph and chain rule

9 points

(a) [2] On $[-1, 1]$, the equal triangles cancel, so $A(1) = 0$. They also cancel on $[1, 3]$. From 3 to 4 the negative triangle has area -1 , so $A(4) = -1$.

Marking: 1 point for each value with signed-area reasoning.

(b) [2] $A' = g$ has signs $-, +, -, +$ separated by zeros $0, 2, 4$. Thus there are local minima $(0, -1)$ and $(4, -1)$, and a local maximum $(2, 1)$.

Marking: 1 point for the three extrema and values; 1 for the sign changes.

(c) [2] $A'' = g' < 0$ on $(1, 3)$ only. Concavity changes at $x = 1, 3$, where the slope of g switches sign.

Marking: 1 point for the interval; 1 for both changes with reasoning.

(d) [3] $J' = e^{A^2}g$ and $J'' = e^{A^2}(2Ag^2 + g')$. At 2, $A = 1$, $g = 0$, and $g' = -2$, so $J'(2) = 0$ and $J''(2) = -2e$.

Marking: 1 point for the first derivative and value; 1 for the second derivative formula; 1 for its value.

Equivalent sound methods earn the same credit.

Free-response solution 4

A differential equation and its interval

Euler method, separable equation, solution interval

9 points

(a) [2] $y_1 = 2 + 0.25(0)(2^2) = 2$ at $x = 0.25$. Then $y_2 = 2 + 0.25(0.25)(2^2) = 2.25$ at $x = 0.5$.

Marking: 1 point for each correctly displayed update.

(b) [2] $y^{-2}dy = x dx$ gives $-1/y = x^2/2 + C$. From $f(0) = 2$, $C = -1/2$, so $f(x) = 2/(1 - x^2)$.

Marking: 1 point for separation and integration; 1 for the initial-condition solution.

(c) [2] The denominator vanishes at $x = \pm 1$. Between those points the formula solves the equation smoothly. The largest such interval containing 0 is $(-1, 1)$.

Marking: 1 point for the singular points; 1 for the correct interval.

(d) [3] $y'' = y^2 + 2xyy' = y^2 + 2x^2y^3 > 0$ on $[0, 0.5]$ since $y > 0$. Successive forward tangent steps lie below this increasing, concave-up solution (the slope xy^2 also increases with positive y). Directly, $f(0.5) = 8/3 > 2.25$.

Marking: 1 point for the second derivative; 1 for positive concavity on the interval; 1 for the justified underestimate, with exact comparison also acceptable.

Equivalent sound methods earn the same credit.

Free-response solution 5

Polar geometry

Polar slope, area and length

9 points

(a) [2] $x' = r' \cos \theta - r \sin \theta$ and $y' = r' \sin \theta + r \cos \theta$, where $r' = -\sin \theta$. At $\pi/2$, $x' = -2$ and $y' = -1$, so $dy/dx = 1/2$.

Marking: 1 point for both derivative formulas; 1 for the slope.

(b) [2] $A = \frac{1}{2} \int_0^{2\pi} (2 + \cos \theta)^2 d\theta = \frac{1}{2}(8\pi + 0 + \pi) = 9\pi/2$.

Marking: 1 point for the polar area integral; 1 for its value.

(c) [3] The curve is outside $r = 2$ when $\cos \theta \geq 0$. Using $-\pi/2 \leq \theta \leq \pi/2$, $A = \frac{1}{2} \int_{-\pi/2}^{\pi/2} [(2 + \cos \theta)^2 - 4] d\theta = 4 + \pi/4$.

Marking: 1 point for the interval; 1 for subtracting squared radii; 1 for the exact area.

(d) [2] $L = \int_0^{2\pi} \sqrt{r^2 + (r')^2} d\theta = \int_0^{2\pi} \sqrt{5 + 4 \cos \theta} d\theta$.

Marking: 1 point for the polar arc-length formula; 1 for the correct integrand and bounds.

Equivalent sound methods earn the same credit.

Free-response solution 6

Series, endpoints, and an error bound

Power series and justified error bounds

9 points

(a) [2] Substitute $u = t^2$ and integrate: $F(x) = x^3/3 - x^5/10 + x^7/21 - \dots = \sum_{n=1}^{\infty} (-1)^{n+1} x^{2n+1} / [n(2n+1)]$.

Marking: 1 point for the three terms; 1 for the general term.

(b) [2] The radius is 1 by the ratio test. At each endpoint the absolute terms are $1/[n(2n+1)] \leq 1/(2n^2)$, so both endpoints converge absolutely by comparison. The interval is $[-1, 1]$.

Marking: 1 point for radius/interior; 1 for both endpoint tests and the interval.

(c) [2] $F(1/2) \approx (1/2)^3/3 - (1/2)^5/10 = 1/24 - 1/320 = 37/960$.

Marking: 1 point for substitution; 1 for the fraction.

(d) [3] The first omitted term is $(1/2)^7/21 = 1/2688$. Terms alternate and their magnitudes decrease to zero, so $|R| \leq 1/2688$. The omitted term is positive, so the approximation is below the true value.

Marking: 1 point for the bound; 1 for the alternating-series conditions; 1 for the direction of error.

Equivalent sound methods earn the same credit.

RIGOROUS CALCULUS PRACTICE

Singapore-inspired

Worked solutions and marking guide

Modeling, optimization and parametric reasoning

60 minutes | 3 structured questions | 30 marks

A graphing calculator may be used for numerical work. Exact answers and proofs are still required where requested.

This is a calculus-focused enrichment paper inspired by selected exam approaches. It is not a full national examination or a claim of syllabus equivalence. Europe has many different examination systems.

How to mark fairly

Award credit for mathematically equivalent methods. The point allocations are Academy practice rubrics, not official exam-board rubrics. Give method credit for sound work even if a later arithmetic error occurs; avoid penalizing the same error twice. Do not award a reasoning point for an unsupported correct number.

Add the marks across the three questions for a score out of 30. Use the topic labels to plan revision. No national grade conversion is implied.

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Structured solution 1

Design an open box

Optimization and sensitivity

10 marks

(a) [2] The dimensions are x , $30 - 2x$, and $20 - 2x$, all positive precisely when $0 < x < 10$. Multiplication gives $V = x(30 - 2x)(20 - 2x) = 4x^3 - 100x^2 + 600x$.

Marking: 1 mark for the domain; 1 for the model and expansion.

(b) [3] $V' = 12x^2 - 200x + 600 = 0$ has roots $(25 \pm 5\sqrt{7})/3$. Only $x_* = (25 - 5\sqrt{7})/3 \approx 3.924$ cm is in $(0, 10)$.

Marking: 1 mark for the derivative; 1 for both algebraic roots; 1 for selecting the physical root.

(c) [3] V' is positive on $(0, x_*)$ and negative on $(x_*, 10)$; the other root exceeds 10. Also the limiting endpoint volumes are zero. Hence x_* is the absolute maximizer, with $V(x_*) \approx 1056.306$ cm³.

Marking: 1 mark for derivative signs; 1 for global/end behavior; 1 for maximum volume.

(d) [2] $V(4) = 1056$ cm³. Percentage loss is $100[V(x_*) - 1056]/V(x_*) \approx 0.029\%$. Because $V'(x_*) = 0$, the first-order change vanishes; the leading small change is quadratic in the displacement.

Marking: 1 mark for the percentage; 1 for the stationary-point explanation.

Equivalent sound methods earn the same credit.

Structured solution 2

Cooling and model interpretation

Differential equations, averages, modeling

10 marks

(a) [3] Separation gives $T - 22 = Ce^{-kt}$. Initially $C = 68$, and $34 = 68e^{-10k}$ implies $k = \ln 2/10 \text{ min}^{-1}$. Thus $T(t) = 22 + 68 \cdot 2^{-t/10}$.

Marking: 1 mark for solving the equation; 1 for the initial condition; 1 for determining k .

(b) [2] $T(25) = 22 + 68 \cdot 2^{-5/2} = 22 + 17/\sqrt{2} \approx 34.021^\circ\text{C}$.

Marking: 1 mark for substitution; 1 for the temperature.

(c) [3] The mean is $\frac{1}{20} \int_0^{20} (22 + 68e^{-kt}) dt = 22 + \frac{68(1-e^{-20k})}{20k} = 22 + \frac{25.5}{\ln 2} \approx 58.789^\circ\text{C}$.

Marking: 1 mark for average-value setup; 1 for integration/exact value; 1 for the decimal.

(d) [2] $8 = 68 \cdot 2^{-t/10}$ gives $t = 10 \ln(8.5)/\ln 2 \approx 30.875$ minutes. For example, room temperature must remain 22°C and the cooling constant must remain applicable.

Marking: 1 mark for time; 1 for a relevant assumption.

Equivalent sound methods earn the same credit.

Structured solution 3

A parametric region

Parametric calculus and geometric integrals

10 marks

(a) [3] $x' = 2t$, $y' = 3t^2 - 1$. The horizontal tangent occurs at $t = 1/\sqrt{3}$, giving $(1/3, -2/(3\sqrt{3}))$. At $t = 0$, $x' = 0$ while $y' = -1 \neq 0$, so the tangent at $(0, 0)$ is vertical (understood as the one-sided endpoint tangent).

Marking: 1 mark for derivatives; 1 for the horizontal point; 1 for the vertical point with justification.

(b) [2] $y \leq 0$ and x increases for $0 \leq t \leq 1$. Area $= -\int y dx = \int_0^1 (t - t^3) 2t dt = 2/3 - 2/5 = 4/15$.

Marking: 1 mark for the parameter integral including dx/dt ; 1 for the value.

(c) [3] $V = \pi \int_0^1 (t^3 - t)^2 (2t) dt = 2\pi \int_0^1 (t^7 - 2t^5 + t^3) dt = 2\pi(1/8 - 1/3 + 1/4) = \pi/12$.

Marking: 1 mark for disks and squared radius; 1 for the dx/dt factor and expansion; 1 for the volume.

(d) [2] $dy/dx = (3t^2 - 1)/(2t)$, so $d^2y/dx^2 = (3t^2 + 1)/(4t^3) > 0$ for $t > 0$. The curve is concave up as a graph of y against x .

Marking: 1 mark for the second derivative; 1 for its sign and interpretation.

Equivalent sound methods earn the same credit.

RIGOROUS CALCULUS PRACTICE

China-inspired

Worked solutions and marking guide

Parameter cases, root counting and inequality proofs

60 minutes | 3 structured questions | 30 marks

No calculator. Give exact answers and complete mathematical justifications.

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How to mark fairly

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Add the marks across the three questions for a score out of 30. Use the topic labels to plan revision. No national grade conversion is implied.

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Edition: September 17, 2026. ChenMathAcademy.org

Structured solution 1

A logarithm with a parameter

Parameter analysis and root counting

10 marks

(a) [3] $f_a' = 1/x - a$ is positive for $0 < x < 1/a$ and negative for $x > 1/a$. The maximum occurs at $x = 1/a$ and equals $a - 1 - \ln a$.

Marking: 1 mark for the derivative and intervals; 1 for the maximum point; 1 for the maximum value.

(b) [3] Let $h(a) = a - 1 - \ln a$. Then $h' = 1 - 1/a$, so h has unique minimum $h(1) = 0$ and is strictly positive otherwise. Thus the maximum of f_a is at most zero exactly when $a = 1$.

Marking: 1 mark for analyzing h ; 1 for its equality case; 1 for both directions of the conclusion.

(c) [2] The function $\ln x - ax$ tends to $-\infty$ as $x \rightarrow 0^+$ and $x \rightarrow \infty$, and has unique maximum $m = -\ln a - 1$ at $1/a$. There are two positive solutions if $b < m$, one if $b = m$, and none if $b > m$.

Marking: 1 mark for the end behavior and maximum; 1 for all three root counts.

(d) [2] For $a = 1$, $f_1(1) = 0$ is the unique maximum and all other values are negative. The graph touches the axis at $(1, 0)$ without changing sign.

Marking: 1 mark for uniqueness; 1 for the no-crossing explanation.

Equivalent sound methods earn the same credit.

Structured solution 2

An absolute maximum for every parameter

Global extrema, parameter cases, exact algebra

10 marks

(a) [2] $g_a' = 3(x^2 - a)$. If $a \leq 0$, g_a is strictly increasing (with only a stationary point at 0 if $a = 0$), so $M = 1 - 3a$ at $x = 1$. If $a \geq 1$, it decreases on the interior, so $M = 3a - 1$ at $x = -1$.

Marking: 1 mark for each range with derivative reasoning.

(b) [3] For $0 < a < 1$, stationary points are $x = \pm\sqrt{a}$. The derivative signs are $+, -, +$, so $-\sqrt{a}$ is a local maximum of value $2a^{3/2}$, and $\sqrt{a}$ is a local minimum. Maximum candidates are $1 - 3a$, $3a - 1$, and $2a^{3/2}$.

Marking: 1 mark for the points; 1 for classification; 1 for all maximum candidates.

(c) [3] Write $u = \sqrt{a} \in (0, 1)$. Then $2a^{3/2} - (3a - 1) = 2u^3 - 3u^2 + 1 = (u - 1)^2(2u + 1) > 0$. Also $2a^{3/2} = 1 - 3a$ is $2u^3 + 3u^2 - 1 = (2u - 1)(u + 1)^2 = 0$, so $a = 1/4$.

Marking: 1 mark for the first factorization and sign; 1 for the equality equation; 1 for $a = 1/4$.

(d) [2] $M(a) = \begin{cases} 1 - 3a, & a \leq 1/4, \\ 2a^{3/2}, & 1/4 < a < 1, \\ 3a - 1, & a \geq 1. \end{cases}$ For $a < 1/4$ the maximizer is 1; for $a = 1/4$ both 1 and $-1/2$

maximize; for $1/4 < a < 1$ it is $-\sqrt{a}$; for $a \geq 1$ it is -1 .

Marking: 1 mark for the complete formula; 1 for all maximizing points, including the tie.

Equivalent sound methods earn the same credit.

Structured solution 3

The sharp constant in an inequality

Inequality proof and optimal constants

10 marks

(a) [2] For $p(x) = x - \ln(1+x)$, $p(0) = 0$ and $p' = x/(1+x) > 0$ for $x > 0$. Thus $p(x) > 0$.

Marking: 1 mark for the derivative; 1 for the monotonicity argument.

(b) [3] $q' = H(x)/x^3$. Direct differentiation gives $H' = -x^2/(1+x)^2 < 0$ and $H(0) = 0$. Therefore $H(x) < 0$ for $x > 0$, and so $q' < 0$.

Marking: 1 mark for q' ; 1 for H' ; 1 for the sign argument.

(c) [3] The proposed inequality is equivalent to $C \leq q(x)$. Since q decreases, its minimum on $(0, 1]$ is $q(1) = 1 - \ln 2$. This value works for every x in the interval, and any larger value fails at $x = 1$. Thus $C = 1 - \ln 2$.

Marking: 1 mark for equivalence; 1 for validity; 1 for sharpness.

(d) [2] L'Hospital gives $\lim q(x) = \lim [x/(1+x)]/(2x) = 1/2$. Strict decrease then gives $1 - \ln 2 < q(x) < 1/2$ for $0 < x < 1$.

Marking: 1 mark for the limit; 1 for both strict bounds.

Equivalent sound methods earn the same credit.

RIGOROUS CALCULUS PRACTICE

Japan-inspired

Worked solutions and marking guide

Integral recurrences, symmetry and geometric proofs

60 minutes | 3 structured questions | 30 marks

No calculator. Give exact answers and complete mathematical justifications.

This is a calculus-focused enrichment paper inspired by selected exam approaches. It is not a full national examination or a claim of syllabus equivalence. Europe has many different examination systems.

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Structured solution 1

An integral recurrence and a limit

Integral recurrence, bounds, sequence limits

10 marks

(a) [3] $I_0 = [\ln(1+x)]_0^1 = \ln 2$. Adding integrands gives $I_n + I_{n+1} = \int_0^1 x^n(1+x)/(1+x)dx = 1/(n+1)$.

Marking: 1 mark for I_0 ; 1 for combining integrands; 1 for the recurrence.

(b) [2] $I_1 = 1 - \ln 2$, $I_2 = 1/2 - I_1 = \ln 2 - 1/2$, and $I_3 = 1/3 - I_2 = 5/6 - \ln 2$.

Marking: 1 mark for the intermediate terms; 1 for I_3 .

(c) [3] $I_n - \frac{1}{2(n+1)} = \int_0^1 \frac{x^n(1-x)}{2(1+x)}dx$. The integrand is nonnegative and at most $x^n(1-x)/2$. Integrating that bound gives $\frac{1}{2}[1/(n+1) - 1/(n+2)] = 1/[2(n+1)(n+2)]$.

Marking: 1 mark for rewriting the difference; 1 for bounding; 1 for integrating the bound.

(d) [2] Multiply the inequality by $n+1$ to obtain $0 \leq (n+1)I_n - 1/2 \leq 1/[2(n+2)]$. The squeeze theorem gives the first limit $1/2$. Then $I_{n+1}/I_n = \frac{(n+2)I_{n+1}}{(n+1)I_n} \frac{n+1}{n+2} \rightarrow 1$.

Marking: 1 mark for each limit with reasoning.

Equivalent sound methods earn the same credit.

Structured solution 2

Oscillation and a volume invariant

Symmetry, substitution, geometric invariants

10 marks

(a) [2] Put $u = nx$. Then $\text{area} = \frac{1}{n} \int_0^{n\pi} |\sin u| du$. Each interval of length π contributes 2, so the total is $2n/n = 2$.

Marking: 1 mark for substitution; 1 for the period argument and area.

(b) [2] $\sin(n\pi - nx) = \sin(n\pi) \cos(nx) - \cos(n\pi) \sin(nx) = (-1)^{n+1} \sin(nx)$. Taking absolute values proves the identity.

Marking: 1 mark for the identity expansion; 1 for the absolute-value conclusion.

(c) [3] The substitution and symmetry give $J_n = \int_0^\pi (\pi - u) |\sin(nu)| du = 2\pi - J_n$. Thus $2J_n = 2\pi$ and $J_n = \pi$.

Marking: 1 mark for the substituted integral; 1 for $2\pi - J_n$; 1 for the exact value.

(d) [3] About the y -axis, $V_n = 2\pi J_n = 2\pi^2$. About the x -axis, disks give $\pi \int_0^\pi \sin^2(nx) dx = \pi [x/2 - \sin(2nx)/(4n)]_0^\pi = \pi^2/2$. Neither depends on positive integer n .

Marking: 1 mark for the shell volume; 1 for the disk integral; 1 for its value and independence.

Equivalent sound methods earn the same credit.

Structured solution 3

Two tangents to a parabola

Tangency, geometric proof, parameter elimination

10 marks

(a) [2] The tangent at a is $L_a(x) = 2ax - a^2$, and at b it is $L_b(x) = 2bx - b^2$. Equality gives $x = (a + b)/2$; substitution gives $y = ab$.

Marking: 1 mark for both tangents; 1 for the intersection.

(b) [2] $x^2 - L_a(x) = (x - a)^2 \geq 0$, with equality only at a . Similarly $x^2 - L_b(x) = (x - b)^2 \geq 0$.

Marking: 1 mark for each squared-difference argument.

(c) [4] Let $m = (a + b)/2$. Since $L_b - L_a = 2(b - a)(x - m)$, the upper tangent is L_a on $[a, m]$ and L_b on $[m, b]$. Thus area $= \int_a^m (x - a)^2 dx + \int_m^b (x - b)^2 dx = \frac{(m-a)^3 + (b-m)^3}{3} = (b - a)^3/12$.

Marking: 1 mark for the boundary switch; 1 for the split integral; 1 for integration; 1 for the formula.

(d) [2] Area 18 gives $(b - a)^3 = 216$, so $b - a = 6$. The midpoint condition gives $a + b = 2$, hence $a = -2$, $b = 4$, and $y = ab = -8$.

Marking: 1 mark for solving for a, b ; 1 for $y = -8$.

Equivalent sound methods earn the same credit.

RIGOROUS CALCULUS PRACTICE

European approaches

Worked solutions and marking guide

Selected A-level and French-style themes: models, sequences and proof

60 minutes | 3 structured questions | 30 marks

A graphing calculator may be used for numerical work. Exact answers and proofs are still required where requested.

This is a calculus-focused enrichment paper inspired by selected exam approaches. It is not a full national examination or a claim of syllabus equivalence. Europe has many different examination systems.

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Structured solution 1

Exponential growth with a carrying capacity

Logistic equation and model evaluation

10 marks

(a) [2] Equilibria are $P = 0$ and $P = 800$. Starting at 100, the solution remains between them and increases because $P' > 0$ there; it approaches 800.

Marking: 1 mark for equilibria; 1 for the increasing solution and limiting population.

(b) [3] $800 \frac{dP}{P(800 - P)} = 0.2 dt$ and $800/[P(800 - P)] = 1/P + 1/(800 - P)$. Integration gives $\ln(P/(800 - P)) = 0.2t + C$. Initially $P/(800 - P) = 1/7$, which rearranges to the stated solution.

Marking: 1 mark for separation and fractions; 1 for integration; 1 for applying the initial condition and solving.

(c) [3] As a function of P , the rate $0.2P - P^2/4000$ is a concave quadratic with maximum at $P = 400$. Its value is 40 individuals/year. From $1 + 7e^{-0.2t} = 2$, the time is $5 \ln 7 \approx 9.730$ years.

Marking: 1 mark for maximizing at 400; 1 for the rate; 1 for the time.

(d) [2] $600 = 800/(1 + 7e^{-0.2t})$ yields $e^{-0.2t} = 1/21$, so $t = 5 \ln 21 \approx 15.223$ years. A changing habitat, migration, or a changing growth parameter could invalidate the fixed-capacity model.

Marking: 1 mark for time; 1 for a relevant model limitation.

Equivalent sound methods earn the same credit.

Structured solution 2

Newton iteration and proof of convergence

Numerical methods, induction, convergence

10 marks

(a) [2] Newton gives $u_{n+1} = u_n - (u_n^2 - 2)/(2u_n) = \frac{1}{2}(u_n + 2/u_n)$. Hence $u_1 = 3/2$ and $u_2 = 17/12$.

Marking: 1 mark for the Newton derivation; 1 for both exact iterates.

(b) [3] $u_{n+1} - \sqrt{2} = (u_n^2 - 2\sqrt{2}u_n + 2)/(2u_n) = (u_n - \sqrt{2})^2/(2u_n)$. The base value $2 > \sqrt{2}$ is positive, and if $u_n > \sqrt{2}$, the identity is strictly positive. Induction proves the claim.

Marking: 1 mark for the identity; 1 for the base case; 1 for the induction step.

(c) [3] $u_{n+1} - u_n = (2 - u_n^2)/(2u_n) < 0$. It is decreasing and bounded below, hence converges to some $L \geq \sqrt{2}$. Taking limits in the recurrence yields $L = (L + 2/L)/2$, so $L^2 = 2$ and $L = \sqrt{2}$.

Marking: 1 mark for decrease; 1 for bounded convergence; 1 for identifying the limit.

(d) [2] Using $u_n > \sqrt{2}$ in the identity from (b) gives the stated upper bound. Once the error is small, the next error is bounded by a constant times its square: quadratic convergence.

Marking: 1 mark for the inequality; 1 for interpreting the squared error.

Equivalent sound methods earn the same credit.

Structured solution 3

A response curve and its parameters

Exponential modeling, improper integral, sensitivity

10 marks

(a) [2] $C' = Ae^{-kt}(1 - kt)$ changes from positive to negative at $t = 1/k$. The peak value is $A/(ke)$.

Marking: 1 mark for the derivative and signs; 1 for both peak expressions.

(b) [2] $1/k = 2$ gives $k = 1/2$. Then $A/(ke) = 12$ gives $A = 6e$.

Marking: 1 mark for each parameter.

(c) [4] Integration by parts gives $\int_0^T Ate^{-kt}dt = \frac{A}{k^2}[1 - (1 + kT)e^{-kT}]$. Since $(1 + kT)e^{-kT} \rightarrow 0$, total response is $A/k^2 = 24e$. At $T = 2$, $kT = 1$, so the fraction is $1 - 2/e$.

Marking: 1 mark for the finite integral; 1 for the improper limit; 1 for the total; 1 for the fraction.

(d) [2] The peak time $1/k$ and peak value $A/(ke)$ each halve. The total A/k^2 is quartered.

Marking: 1 mark for both peak changes; 1 for the total change.

Equivalent sound methods earn the same credit.