

RIGOROUS CALCULUS PRACTICE

AP Calculus AB

Worked solutions and marking guide

Foundations through differential equations

Section	Questions / time	Calculator
MCQ Part A	29 / 62 minutes	Not permitted
MCQ Part B	13 / 38 minutes	Graphing required
FRQ Part A	2 / 30 minutes	Graphing required
FRQ Part B	4 / 60 minutes	Not permitted

Total working time: **190 minutes**. MCQ and FRQ each carry 50% of this mock's weighted practice score.

How to mark fairly

Award credit for mathematically equivalent methods. The point allocations are Academy practice rubrics, not official exam-board rubrics. Give method credit for sound work even if a later arithmetic error occurs; avoid penalizing the same error twice. Do not award a reasoning point for an unsupported correct number.

Practice score: Let m be the number of correct MCQs out of 42, and r the total FRQ points out of 54. Compute $50(m/42) + 50(r/54)$ out of 100. There is no penalty for an incorrect MCQ. This score is not an AP 1–5 prediction; no official grade boundaries are supplied.

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Quick answer key

Question	Answer	Question	Answer	Question	Answer
1	D	15	D	29	D
2	A	16	D	30	B
3	C	17	C	31	C
4	A	18	C	32	B
5	D	19	D	33	C
6	A	20	D	34	D
7	B	21	D	35	A
8	D	22	B	36	B
9	B	23	C	37	D
10	C	24	D	38	B
11	C	25	B	39	C
12	A	26	D	40	A
13	A	27	A	41	C
14	D	28	B	42	D

Diagnose before you repeat

For each missed question, record the topic, the incorrect step, and the idea that fixes it. Rework the question without reading the explanation. Then return to the relevant Academy video and its lesson worksheet.

Multiple-choice worked solutions

1. Answer D

Limits

Correct choice: $-\frac{9}{2}$

The quotient has form $0/0$. Three applications of L'Hospital give $\lim[-27 \cos(3x)/6] = -9/2$. Each intermediate quotient has form $0/0$.

2. Answer A

Continuity and differentiability

Correct choice: $(2, 2)$

Continuity requires $k = 2$. Then $f'(0) = \lim_{h \rightarrow 0} (e^{2h} - 1 - 2h)/h^2 = 2$ by two applications of L'Hospital.

3. Answer C

Inverse functions

Correct choice: $\frac{6}{5}$

Chain rule: $h'(x) = 2x/f'(f^{-1}(x^2))$. At $x = 3$, the inverse input is 2, so $h'(3) = 6/5$.

4. Answer A

Implicit differentiation

Correct choice: $-\frac{42}{125}$

First $2x + y + (x + 2y)y' = 0$, giving $y' = -4/5$. Differentiate again: $2 + 2y' + 2(y')^2 + (x + 2y)y'' = 0$. Substitute to obtain $y'' = -42/125$.

Multiple-choice worked solutions

5. Answer D

Chain rule

Correct choice: $y = 1 - (x - \pi)$

$f(\pi) = 1$ and $f'(x) = e^{\sin x} \cos x$, so the slope at π is -1 .

6. Answer A

Logarithmic differentiation

Correct choice: $4(1 + \ln 2)$

Logarithmic differentiation gives $\ln f = x \ln x$, so $f'/f = 1 + \ln x$ and $f'(2) = 4(1 + \ln 2)$.

7. Answer B

Chain and quotient rules

Correct choice: $-\frac{9}{4}$

$H' = g'[gf'(g) - f(g)]/g^2$. Substitute: $(-3)(2 \cdot 4 - 5)/4 = -9/4$.

8. Answer D

FTC and limits

Correct choice: 14

The numerator is zero at $h = 0$ and its derivative is $2f(3 + 2h)$ by FTC and chain rule. The limit is $2f(3) = 14$.

Multiple-choice worked solutions

9. Answer B

FTC

Correct choice: $2 \ln 2$

$F'(x) = 2x \ln(1 + x^4)$, hence $F'(1) = 2 \ln 2$.

10. Answer C

Riemann sums

Correct choice: $\frac{7}{3}$

This is a right-endpoint Riemann sum for $\int_1^2 x^2 dx = [x^3/3]_1^2 = 7/3$.

11. Answer C

MVT hypotheses

Correct choice: f is not differentiable at 0.

The function is continuous on the closed interval, but has a corner at 0. In fact the secant slope is 0, whereas every existing derivative on the interior is -1 or 1 .

12. Answer A

Rolle theorem

Correct choice: $c = -1, 1$

The polynomial is continuous and differentiable, and its endpoint values are both 0. Solve $f'(c) = 3c^2 - 3 = 0$: $c = \pm 1$, both interior.

Multiple-choice worked solutions

13. Answer A

Derivative graph

Correct choice: 3

$f' > 0$ on $(-2, 2)$ and $f' < 0$ on $(2, 4)$, so the maximum occurs at 2. The area under f' from 0 to 2 is a triangle of area 2, so $f(2) = 1 + 2 = 3$.

14. Answer D

Derivative sign analysis

Correct choice: f has a local minimum at $x = -2$ and no extremum at $x = 1$.

The squared factor does not change sign. The derivative changes from negative to positive at -2 , but is positive on both sides of 1.

15. Answer D

Absolute value integrals

Correct choice: 2

Split at 1: $\int_0^1 (1 - x^2) dx = 2/3$ and $\int_1^2 (x^2 - 1) dx = 4/3$. Their sum is 2.

16. Answer D

Average value

Correct choice: $\frac{1}{2(e-1)}$

Average value is $(e-1)^{-1} \int_1^e (\ln x)/x dx$. An antiderivative is $(\ln x)^2/2$, giving $1/[2(e-1)]$.

Multiple-choice worked solutions

17. Answer C

MVT and concavity

Correct choice: $f'(0) < 2 < f'(4)$

The secant slope is 2. MVT gives an interior c with $f'(c) = 2$, and $f'' > 0$ makes f' strictly increasing, giving the strict endpoint inequalities.

18. Answer C

Cross-section volumes

Correct choice: $\frac{1}{30}$

Area of a square slice is $(\sqrt{x} - x)^2 = x - 2x^{3/2} + x^2$. Integrate from 0 to 1: $1/2 - 4/5 + 1/3 = 1/30$.

19. Answer D

Washer method

Correct choice: $\frac{8\pi}{15}$

Outer radius $2 - x^2$ and inner radius $2 - x$ give $V = \pi \int_0^1 [(2 - x^2)^2 - (2 - x)^2] dx = \pi(2 - 5/3 + 1/5) = 8\pi/15$.

20. Answer D

Separation of variables

Correct choice: $y = -\frac{2}{x^2+2}$

Separate: $y^{-2} dy = x dx$. Then $-1/y = x^2/2 + C$ and $C = 1$, giving $y = -2/(x^2 + 2)$.

Multiple-choice worked solutions

21. Answer D

Differential-equation analysis

Correct choice: increasing and concave up.

At the point, $y' = 2 > 0$. Differentiating yields $y'' = (3 - 2y)y' = 2 > 0$.

22. Answer B

Linearization

Correct choice: 2.05, an overestimate.

$L(x) = 2 + (x - 4)/4$, giving 2.05. Since $f''(x) < 0$, the graph lies below this tangent line.

23. Answer C

Related rates

Correct choice: $16\pi \text{ cm}^3/\text{s}$

$V = \pi h^3/27$. Thus $dV/dt = (\pi h^2/9)(dh/dt)$; at the instant this equals 16π .

24. Answer D

Optimization

Correct choice: $3\sqrt{5}$

With height $30/x$, cost $C = 4x + 180/x$. Set $C' = 4 - 180/x^2 = 0$, giving $x = \sqrt{45} = 3\sqrt{5}$. Since $C'' = 360/x^3 > 0$, this is the minimum.

Multiple-choice worked solutions

25. Answer B

Inverse trig integrals

Correct choice: $\frac{\pi}{4}$

Use $u = 2x$. The integral becomes $\int_0^1 (1 + u^2)^{-1} du = \arctan 1 - \arctan 0 = \pi/4$.

26. Answer D

Substitution

Correct choice: $\frac{1}{4}$

With $u = \cos x$, $du = -\sin x \, dx$ and bounds 1 to 0. The value is $\int_0^1 u^3 du = 1/4$.

27. Answer A

Limits at infinity

Correct choice: $\frac{1}{3}$

Rationalize: $2x/(\sqrt{9x^2 + 2x} + 3x) = 2/(\sqrt{9 + 2/x} + 3) \rightarrow 1/3$.

28. Answer B

Differentiability versus continuity

Correct choice: $f'(0) = 0$, but f' is not continuous at 0.

The difference quotient is $h \sin(1/h) \rightarrow 0$. For $x \neq 0$, $f'(x) = 2x \sin(1/x) - \cos(1/x)$; its oscillating cosine term prevents a limit at 0.

Multiple-choice worked solutions

29. Answer D

Parameter analysis

Correct choice: $k \leq 0$

$f'(t) = 3(x^2 - k)$. For $k < 0$ it is positive everywhere; for $k = 0$, x^3 remains strictly increasing. If $k > 0$, the derivative is negative near 0.

30. Answer B

Numerical zeros

Correct choice: 0.567

Use a numerical zero of $e^{-x} - x$. Its derivative $-e^{-x} - 1 < 0$ and endpoint signs differ, so the root is unique. Numerically, the answer is 0.567143, which rounds to 0.567.

31. Answer C

Area between curves

Correct choice: 0.272

Solve $e^{-a} = a$, giving $a \approx 0.567143290$. Area $= \int_0^a (e^{-x} - x)dx = 1 - a - a^2/2$. Numerically, the answer is 0.272031, which rounds to 0.272.

32. Answer B

Motion

Correct choice: 2.596

Find the nonzero root of $2 \sin t - 0.4t = 0$. Velocity changes from positive to negative there; $t = 0$ is only an endpoint. Numerically, the answer is 2.595739, which rounds to 2.596.

Multiple-choice worked solutions

33. Answer C

Total distance

Correct choice: 9.636

Velocity zeros in the interval are 0.254102 and 1.860806. Integrate $|v|$, splitting at both roots, and add positive distances. Numerically, the answer is 9.636005, which rounds to 9.636.

34. Answer D

Accumulation model

Correct choice: 31.494

Use initial amount plus accumulated net rate: $30 + \int_0^4 [1 + 2 \sin(t^2) - 0.5t] dt$. Numerically, the answer is 31.494268, which rounds to 31.494.

35. Answer A

Average value

Correct choice: 0.441

The average value is $\frac{1}{2} \int_0^2 e^{-x^2} dx$. Use numerical integration and divide by the interval length. Numerically, the answer is 0.441041, which rounds to 0.441.

36. Answer B

MVT numerical solution

Correct choice: 2.138

The secant slope is $\ln(10)/3$. Solve $2c/(1+c^2) = \ln(10)/3$; both positive roots lie in $(0, 3)$, and select the larger. Numerically, the answer is 2.138051, which rounds to 2.138.

Multiple-choice worked solutions

37. Answer D

Normal line

Correct choice: -1.557

Solve $\ln a = 2 - a$. The tangent slope is $1/a$, so the normal slope is its negative reciprocal, $-a$. Numerically, the answer is -1.557146 , which rounds to -1.557 .

38. Answer B

Differential equations

Correct choice: 2.541

Separate and integrate: $y^2/2 = -\cos x + C$. The initial value gives $y^2 = 6 - 2\cos x$; choose the positive branch. Numerically, the answer is 2.540552 , which rounds to 2.541 .

39. Answer C

Numerical volume

Correct choice: 1.969

Use disks with squared radius: $V = \pi \int_0^2 (e^{-x^2})^2 dx = \pi \int_0^2 e^{-2x^2} dx$. Numerically, the answer is 1.968577 , which rounds to 1.969 .

40. Answer A

Tabular integration

Correct choice: 18.500

Use the actual widths: $1(2 + 5)/2 + 2(5 + 4)/2 + 1(4 + 8)/2 = 18.5$ liters. Numerically, the answer is 18.500000 , which rounds to 18.500 .

Multiple-choice worked solutions

41. Answer C

Implicit slope

Correct choice: -0.368

First solve $2e^y + y^2 = 5$, obtaining $y \approx 0.784825523$. Implicit differentiation gives $y' = -e^y/(xe^y + 2y)$. Numerically, the answer is -0.368179 , which rounds to -0.368 .

42. Answer D

Accumulation inverse problem

Correct choice: 1.235

Solve $\int_0^t [2 + \sin(u^2)] du = 3$ numerically. The integrand is at least 1, so the accumulated amount is strictly increasing and the time is unique. Numerically, the answer is 1.234677 , which rounds to 1.235 .

Free-response solution 1

A storage tank

Accumulation, rates, global extrema

9 points

(a) [2] The total inflow is $\int_0^8 10e^{-0.2t} dt = 50(1 - e^{-1.6}) \approx 39.905$ liters.

Marking: 1 point for the inflow integral; 1 for the value with units.

(b) [2] $W(8) = 20 + \int_0^8 [10e^{-0.2t} - 2 - 0.4t] dt \approx 31.105$ liters. Initial amount must be included.

Marking: 1 point for initial amount plus net accumulation; 1 for the value.

(c) [2] $W'(4) = 10e^{-0.8} - 3.6 \approx 0.893 > 0$, so the amount increases. $W''(4) = -2e^{-0.8} - 0.4 < 0$, so its rate of change decreases.

Marking: 1 point for each correctly justified conclusion.

(d) [3] $W'(t) = 0$ gives $t = c \approx 4.722$ minutes. Because $W''(t) = -2e^{-0.2t} - 0.4 < 0$, the net rate decreases strictly, is positive before c , and negative after c . Therefore the unique absolute maximum is $W(c) \approx 36.651$ liters.

Marking: 1 point for the critical time; 1 for the sign-change/global justification; 1 for the maximum amount.

Equivalent sound methods earn the same credit.

Free-response solution 2

Motion on a line

Motion, numerical integration, speed

9 points

(a) [2] $x(6) = 2 + \int_0^6 v(t)dt \approx 2.194$ meters. Average velocity is $[x(6) - 2]/6 \approx 0.032$ m/s.

Marking: 1 point for position with its integral; 1 for average velocity.

(b) [2] The two roots are $t \approx 0.104$ and 2.878 . The velocity signs are negative, positive, negative on the three resulting intervals. Each root is a change of direction.

Marking: 1 point for both roots; 1 for the signs and direction-change conclusion.

(c) [2] $D = -\int_0^{0.103702982} v(t)dt + \int_{0.103702982}^{2.877577344} v(t)dt - \int_{2.877577344}^6 v(t)dt \approx 1.646$ meters. Equivalently integrate $|v|$.

Marking: 1 point for an absolute-value or correctly split integral; 1 for the distance.

(d) [3] $a(t) = e^{-t/3}[\cos t - \frac{1}{3}\sin t]$. At $t = 4$, $v(4) \approx -0.299 < 0$ and $a(4) \approx -0.106 < 0$. Velocity and acceleration have the same sign, so speed increases.

Marking: 1 point for the acceleration formula; 1 for both signs; 1 for the speed conclusion.

Equivalent sound methods earn the same credit.

Free-response solution 3

Read a derivative graph

Graphical analysis and chain rule

9 points

(a) [2] $f(-2) = 1 - 2 = -1$. From 0 to 2 the signed area is 2; from 2 to 4 it is $-\pi/2$. Thus $f(4) = 3 - \pi/2$.

Marking: 1 point for each exact value with signed areas.

(b) [2] $f' > 0$ on $(-2, 2)$ and $f' < 0$ on $(2, 4)$, so the maximum is $f(2) = 3$. Compare the endpoints: $f(-2) = -1 < f(4) = 3 - \pi/2$, giving minimum -1 .

Marking: 1 point for the maximum with signs; 1 for the endpoint comparison and minimum.

(c) [2] f' increases on $(-2, 0)$ and $(3, 4)$, so f is concave up there. The derivative changes from increasing to decreasing at 0, and from decreasing to increasing at 3, so inflection points occur at $x = 0, 3$. There is no concavity change at 2.

Marking: 1 point for both intervals; 1 for both inflection coordinates with reasoning.

(d) [3] $h'(x) = 2xf'(x^2)$ and $h''(x) = 2f'(x^2) + 4x^2f''(x^2)$. The line segment gives $f'(1) = 1$ and $f''(1) = -1$, so $h'(1) = 2$ and $h''(1) = -2$.

Marking: 1 point for the first derivative and value; 1 for the second-derivative formula; 1 for its value.

Equivalent sound methods earn the same credit.

Free-response solution 4

A separable differential equation

Differential equations and tangent error

9 points

(a) [2] $f'(0) = 1$, so $L(x) = 3 + x$ and $f(0.1) \approx 3.1$.

Marking: 1 point for the line; 1 for the approximation.

(b) [2] $y'' = 2(y - 2) + (2x + 1)y'$. At $(0, 3)$, this gives $f''(0) = 2 + 1 = 3$.

Marking: 1 point for the derivative formula; 1 for the evaluation.

(c) [3] $dy/(y-2) = (2x+1)dx$ gives $\ln|y-2| = x^2+x+C$. Since $y(0)-2 = 1$, the solution is $f(x) = 2+e^{x^2+x}$.

Marking: 1 point for separation; 1 for integration; 1 for the particular solution.

(d) [2] $f''(x) = [2 + (2x + 1)^2]e^{x^2+x} > 0$ throughout $[0, 0.1]$. The curve is strictly concave up and lies above its tangent, so 3.1 is an underestimate.

Marking: 1 point for positivity on the interval; 1 for the justified underestimate.

Equivalent sound methods earn the same credit.

Free-response solution 5

One region, three solids

Area, cross sections, axis changes

9 points

(a) [2] Intersections satisfy $x^2 = 2x$, giving $x = 0, 2$. Area $= \int_0^2 (2x - x^2) dx = [x^2 - x^3/3]_0^2 = 4/3$.

Marking: 1 point for the integral and bounds; 1 for the area.

(b) [2] The side is $2x - x^2$, so $V = \int_0^2 (2x - x^2)^2 dx = [4x^3/3 - x^4 + x^5/5]_0^2 = 16/15$.

Marking: 1 point for squared cross-sectional area; 1 for the volume.

(c) [2] Outer radius is $4 - x^2$ and inner radius is $4 - 2x$. Thus $V = \pi \int_0^2 [(4 - x^2)^2 - (4 - 2x)^2] dx$.

Marking: 1 point for both radii; 1 for the correct integral.

(d) [3] At height y , the left and right boundaries are $x = y/2$ and $x = \sqrt{y}$, for $0 \leq y \leq 4$. Therefore $V = \pi \int_0^4 [y - y^2/4] dy = \pi [y^2/2 - y^3/12]_0^4 = 8\pi/3$.

Marking: 1 point for boundaries and limits; 1 for the washer integral; 1 for the volume.

Equivalent sound methods earn the same credit.

Free-response solution 6

Function values and theorems

Table, FTC, inverse and MVT

9 points

(a) [2] $H'(x) = 2xf(x^2)$, so $H'(1) = 4$. Also $H''(x) = 2f(x^2) + 4x^2f'(x^2)$, giving $H''(1) = 4 - 4 = 0$.

Marking: 1 point for each value with its derivative formula.

(b) [2] f is continuous on $[2, 3]$ and differentiable on $(2, 3)$. The Mean Value Theorem gives $f'(c) = [f(3) - f(2)]/(3 - 2) = 4$.

Marking: 1 point for the hypotheses; 1 for the theorem and secant calculation.

(c) [2] Because $f'(2) = 2 \neq 0$ and f' is continuous, f is strictly increasing near 2 and has a differentiable local inverse. Then $g'(0) = 1/f'(2) = 1/2$.

Marking: 1 point for local invertibility reasoning; 1 for the derivative.

(d) [3] By linearity and FTC, $\int_0^3 [2f + f'] dx = 2 \int_0^3 f dx + f(3) - f(0) = 2(6) + 4 - 1 = 15$.

Marking: 1 point for linearity; 1 for FTC endpoint difference; 1 for the value.

Equivalent sound methods earn the same credit.