

RIGOROUS CALCULUS PRACTICE

AP Calculus AB

Full-length mock | 2027 structure

Foundations through differential equations

Section	Questions / time	Calculator
MCQ Part A	29 / 62 minutes	Not permitted
MCQ Part B	13 / 38 minutes	Graphing required
FRQ Part A	2 / 30 minutes	Graphing required
FRQ Part B	4 / 60 minutes	Not permitted

Total working time: **190 minutes**. MCQ and FRQ each carry 50% of this mock's weighted practice score.

Before you begin

Write your name and date below. Keep the separate solutions file closed until you finish. Show equations, reasoning, units and required justifications. Use radian mode. Unless an exact answer is requested, round final numerical answers to three decimal places and retain unrounded values during calculations.

Choose exactly one answer for each MCQ. Use the answer sheet at the back of Section I. Time the four parts separately and keep the calculator closed in the two no-calculator parts. These printable pages do not simulate the Bluebook interface.

Name: _____ Date: _____

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Section I | Part A**No calculator**

29 questions / 62 minutes

1. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(3x) - 3x}{x^3}$.

(A) $-\frac{3}{2}$

(B) 0

(C) $\frac{9}{2}$

(D) $-\frac{9}{2}$

2. For $x \neq 0$, let $f(x) = (e^{2x} - 1)/x$, and define $f(0) = k$. Which pair $(k, f'(0))$ makes f differentiable at 0?

(A) (2, 2)

(B) (1, 2)

(C) (0, 2)

(D) (2, 4)

3. A differentiable one-to-one function satisfies $f(2) = 9$ and $f'(2) = 5$. If $h(x) = f^{-1}(x^2)$, find $h'(3)$.

(A) $\frac{3}{5}$

(B) $\frac{1}{5}$

(C) $\frac{6}{5}$

(D) 30

Section I | Part A**No calculator**

29 questions / 62 minutes

4. The curve $x^2 + xy + y^2 = 7$ passes through $(1, 2)$. What is d^2y/dx^2 there?

(A) $-\frac{42}{125}$

(B) $\frac{42}{125}$

(C) $-\frac{4}{5}$

(D) $-\frac{2}{5}$

5. Let $f(x) = e^{\sin x}$. Which is the tangent line at $x = \pi$?

(A) $y = 1 + (x - \pi)$

(B) $y = e^\pi(x - \pi)$

(C) $y = -(x - \pi)$

(D) $y = 1 - (x - \pi)$

6. If $f(x) = x^x$ for $x > 0$, find $f'(2)$.

(A) $4(1 + \ln 2)$

(B) 8

(C) $2(1 + \ln 2)$

(D) $4 \ln 2$

Section I | Part A**No calculator**

29 questions / 62 minutes

7. Suppose $g(1) = 2$, $g'(1) = -3$, $f(2) = 5$, and $f'(2) = 4$. For $H(x) = f(g(x))/g(x)$, find $H'(1)$.

- (A) $-\frac{39}{4}$
- (B) $-\frac{9}{4}$
- (C) -6
- (D) $\frac{9}{4}$

8. A continuous function f has $f(3) = 7$. Evaluate $\lim_{h \rightarrow 0} \frac{1}{h} \int_3^{3+2h} f(t) dt$.

- (A) 7
- (B) $\frac{7}{2}$
- (C) 0
- (D) 14

9. For $F(x) = \int_1^{x^2} \ln(1+t^2) dt$, find $F'(1)$.

- (A) $\ln 2$
- (B) $2 \ln 2$
- (C) $4 \ln 2$
- (D) 2

Section I | Part A**No calculator**

29 questions / 62 minutes

10. Evaluate $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n} \left(1 + \frac{k}{n}\right)^2$.

(A) $\frac{1}{3}$

(B) 3

(C) $\frac{7}{3}$

(D) $\frac{8}{3}$

11. For $f(x) = |x|$ on $[-1, 1]$, why can the Mean Value Theorem not be applied?

(A) $f(-1) \neq f(1)$.

(B) The average rate of change is undefined.

(C) f is not differentiable at 0.(D) f is not continuous at 0.

12. For $f(x) = x^3 - 3x$ on $[-\sqrt{3}, \sqrt{3}]$, which values of c satisfy the conclusion of Rolle's theorem?

(A) $c = -1, 1$

(B) $c = 1$ only

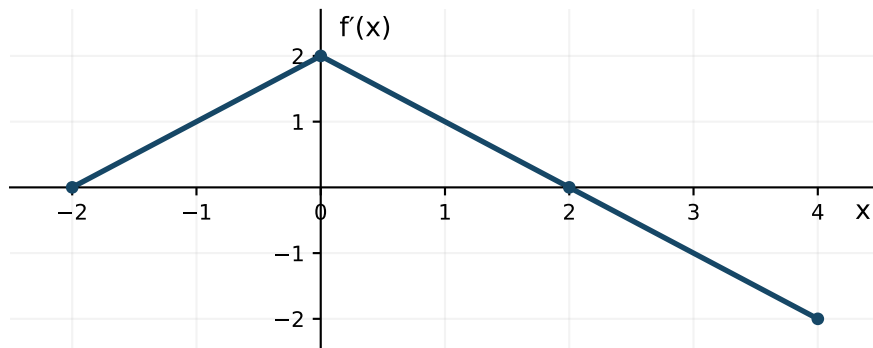
(C) $c = 0$

(D) $c = -\sqrt{3}, \sqrt{3}$

Section I | Part A**No calculator**

29 questions / 62 minutes

13. The graph shown is f' . Its line segments join $(-2, 0)$, $(0, 2)$, $(2, 0)$, and $(4, -2)$. If $f(0) = 1$, what is the absolute maximum value of f on $[-2, 4]$?



- (A) 3
- (B) 5
- (C) 2
- (D) 1

14. If $f'(x) = (x - 1)^2(x + 2)$, which statement is true?

- (A) f has a local maximum at 1.
- (B) f has local minima at both -2 and 1 .
- (C) f has a local maximum at -2 .
- (D) f has a local minimum at $x = -2$ and no extremum at $x = 1$.

Section I | Part A**No calculator**

29 questions / 62 minutes

15. Evaluate $\int_0^2 |x^2 - 1| dx$.

- (A) $\frac{4}{3}$
- (B) $\frac{2}{3}$
- (C) $\frac{8}{3}$
- (D) 2

16. What is the average value of $f(x) = \ln x/x$ on $[1, e]$?

- (A) $\frac{1}{2}$
- (B) $\frac{e-1}{2}$
- (C) $\frac{1}{e-1}$
- (D) $\frac{1}{2(e-1)}$

17. A twice-differentiable function has $f(0) = 2$, $f(4) = 10$, and $f''(x) > 0$ on $(0, 4)$. Which must hold?

- (A) $f'(0) > 2 > f'(4)$
- (B) $f'(x) = 2$ throughout
- (C) $f'(0) < 2 < f'(4)$
- (D) $f'(2) = 2$

Section I | Part A**No calculator**

29 questions / 62 minutes

18. A solid has base between $y = \sqrt{x}$ and $y = x$ on $0 \leq x \leq 1$. Cross sections perpendicular to the x -axis are squares. Find its volume.

- (A) $\frac{1}{6}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{30}$
- (D) $\frac{1}{15}$

19. The region between $y = x$ and $y = x^2$, $0 \leq x \leq 1$, is revolved about $y = 2$. Find the volume.

- (A) $\frac{2\pi}{15}$
- (B) $\frac{4\pi}{15}$
- (C) $\frac{\pi}{6}$
- (D) $\frac{8\pi}{15}$

20. The solution of $y' = xy^2$ with $y(0) = -1$ satisfies which equation?

- (A) $y = -1 + \frac{x^2}{2}$
- (B) $y = -e^{x^2/2}$
- (C) $y = \frac{2}{x^2-2}$
- (D) $y = -\frac{2}{x^2+2}$

Section I | Part A**No calculator**

29 questions / 62 minutes

21. A solution of $y' = y(3 - y)$ passes through $(0, 1)$. At that point, the solution is

- (A) decreasing and concave down.
- (B) decreasing and concave up.
- (C) increasing and concave down.
- (D) increasing and concave up.

22. Using the tangent line to $f(x) = \sqrt{x}$ at $x = 4$, the estimate of $\sqrt{4.2}$ is

- (A) 2.10, an overestimate.
- (B) 2.05, an overestimate.
- (C) 2.025, an underestimate.
- (D) 2.05, an underestimate.

23. Water fills a cone with radius always one-third of its water depth h . When $h = 6$ cm and $dh/dt = 4$ cm/s, find dV/dt .

- (A) $\frac{16\pi}{3}$ cm³/s
- (B) 48π cm³/s
- (C) 16π cm³/s
- (D) 144π cm³/s

Section I | Part A**No calculator**

29 questions / 62 minutes

24. A rectangle has area 30. Its horizontal edges cost 2 per unit length and its vertical edges cost 3 per unit length. Which horizontal length x minimizes the total cost?

- (A) $\frac{3\sqrt{5}}{2}$
- (B) $2\sqrt{5}$
- (C) $\sqrt{30}$
- (D) $3\sqrt{5}$

25. Evaluate $\int_0^{1/2} \frac{2}{1+4x^2} dx$.

- (A) $\frac{\pi}{2}$
- (B) $\frac{\pi}{4}$
- (C) $\frac{\pi}{8}$
- (D) $\ln 2$

26. Evaluate $\int_0^{\pi/2} \sin x \cos^3 x \, dx$.

- (A) 1
- (B) $\frac{3}{4}$
- (C) $\frac{1}{2}$
- (D) $\frac{1}{4}$

Section I | Part A**No calculator**

29 questions / 62 minutes

27. Evaluate $\lim_{x \rightarrow +\infty} (\sqrt{9x^2 + 2x} - 3x)$.

- (A) $\frac{1}{3}$
- (B) $+\infty$
- (C) $\frac{2}{3}$
- (D) 0

28. Define $f(0) = 0$ and $f(x) = x^2 \sin(1/x)$ for $x \neq 0$. Which statement is correct?

- (A) $f'(0) = 1$.
- (B) $f'(0) = 0$, but f' is not continuous at 0.
- (C) $f'(0)$ does not exist.
- (D) f' is continuous at 0.

29. For which real parameters k is $f(x) = x^3 - 3kx$ increasing on all of $\mathbb{R}$?

- (A) $k \geq 0$
- (B) $k < 0$ only
- (C) $k = 0$ only
- (D) $k \leq 0$

STOP. End of this part. Check your responses before moving to the next timed part.

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

30. Find the unique solution of $e^{-x} = x$ on $[0, 1]$.

- (A) 0.739
- (B) 0.567
- (C) 0.432
- (D) 0.693

31. The curves $y = e^{-x}$ and $y = x$ intersect at $x = a > 0$. Find the area enclosed by these curves and the y -axis.

- (A) 0.136
- (B) 0.472
- (C) 0.272
- (D) 0.728

32. A particle has velocity $v(t) = 2 \sin t - 0.4t$ for $0 \leq t \leq 5$. At what positive time does it change direction?

- (A) 3.096
- (B) 2.596
- (C) 2.096
- (D) 3.142

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

33. A particle has velocity $v(t) = t^3 - 4t + 1$ on $[0, 3]$. Find the total distance traveled.

- (A) 11.636
- (B) 5.250
- (C) 9.636
- (D) 4.818

34. A tank starts with 30 liters. Inflow is $5 + 2\sin(t^2)$ L/min and outflow is $4 + 0.5t$ L/min. How much water is in the tank at $t = 4$?

- (A) 35.494
- (B) 1.494
- (C) 51.494
- (D) 31.494

35. Find the average value of e^{-x^2} on $[0, 2]$.

- (A) 0.441
- (B) 0.221
- (C) 0.882
- (D) 0.632

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

36. For $f(x) = \ln(1 + x^2)$ on $[0, 3]$, find the larger number c satisfying the Mean Value Theorem conclusion.

- (A) 0.468
- (B) 2.138
- (C) 1.000
- (D) 3.000

37. The curve $y = \ln x$ intersects $y = 2 - x$ at $x = a$. Find the slope of the normal to $y = \ln x$ at that point.

- (A) 1.557
- (B) -0.642
- (C) 0.642
- (D) -1.557

38. A solution of $dy/dx = \sin x/y$ satisfies $y(0) = 2$. Find $y(1.8)$.

- (A) 6.454
- (B) 2.541
- (C) 1.270
- (D) 1.541

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

39. The region below $y = e^{-x^2}$ and above the x -axis on $[0, 2]$ is rotated about the x -axis. Find the volume.

- (A) 3.937
- (B) 0.627
- (C) 1.969
- (D) 2.771

40. The table gives a flow rate $r(t)$ in L/min. Estimate $\int_0^4 r(t)dt$ by the trapezoidal rule on the three listed subintervals.

t	0	1	3	4
$r(t)$	2	5	4	8

- (A) 18.500
- (B) 19.500
- (C) 17.000
- (D) 21.000

41. The curve $xe^y + y^2 = 5$ has a point with $x = 2$ and $y > 0$. Find dy/dx at that point.

- (A) -0.184
- (B) -2.716
- (C) -0.368
- (D) 0.368

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

42. A quantity starts at 0 and increases at rate $2 + \sin(t^2)$ units/min. When does it first reach 3 units?

- (A) 1.500
- (B) 1.735
- (C) 0.935
- (D) 1.235

STOP. End of this part. Check your responses before moving to the next timed part.

Multiple-choice answer sheet

Mark one response per question.

- | | | |
|---|---|---|
| 1. ☐ A ☐ B ☐ C ☐ D | 15. ☐ A ☐ B ☐ C ☐ D | 29. ☐ A ☐ B ☐ C ☐ D |
| 2. ☐ A ☐ B ☐ C ☐ D | 16. ☐ A ☐ B ☐ C ☐ D | 30. ☐ A ☐ B ☐ C ☐ D |
| 3. ☐ A ☐ B ☐ C ☐ D | 17. ☐ A ☐ B ☐ C ☐ D | 31. ☐ A ☐ B ☐ C ☐ D |
| 4. ☐ A ☐ B ☐ C ☐ D | 18. ☐ A ☐ B ☐ C ☐ D | 32. ☐ A ☐ B ☐ C ☐ D |
| 5. ☐ A ☐ B ☐ C ☐ D | 19. ☐ A ☐ B ☐ C ☐ D | 33. ☐ A ☐ B ☐ C ☐ D |
| 6. ☐ A ☐ B ☐ C ☐ D | 20. ☐ A ☐ B ☐ C ☐ D | 34. ☐ A ☐ B ☐ C ☐ D |
| 7. ☐ A ☐ B ☐ C ☐ D | 21. ☐ A ☐ B ☐ C ☐ D | 35. ☐ A ☐ B ☐ C ☐ D |
| 8. ☐ A ☐ B ☐ C ☐ D | 22. ☐ A ☐ B ☐ C ☐ D | 36. ☐ A ☐ B ☐ C ☐ D |
| 9. ☐ A ☐ B ☐ C ☐ D | 23. ☐ A ☐ B ☐ C ☐ D | 37. ☐ A ☐ B ☐ C ☐ D |
| 10. ☐ A ☐ B ☐ C ☐ D | 24. ☐ A ☐ B ☐ C ☐ D | 38. ☐ A ☐ B ☐ C ☐ D |
| 11. ☐ A ☐ B ☐ C ☐ D | 25. ☐ A ☐ B ☐ C ☐ D | 39. ☐ A ☐ B ☐ C ☐ D |
| 12. ☐ A ☐ B ☐ C ☐ D | 26. ☐ A ☐ B ☐ C ☐ D | 40. ☐ A ☐ B ☐ C ☐ D |
| 13. ☐ A ☐ B ☐ C ☐ D | 27. ☐ A ☐ B ☐ C ☐ D | 41. ☐ A ☐ B ☐ C ☐ D |
| 14. ☐ A ☐ B ☐ C ☐ D | 28. ☐ A ☐ B ☐ C ☐ D | 42. ☐ A ☐ B ☐ C ☐ D |

End of Section I.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 1: A storage tank

9 points

A tank contains 20 liters at $t = 0$. Water enters at $r(t) = 10e^{-0.2t}$ liters per minute and leaves at $s(t) = 2 + 0.4t$ liters per minute, for $0 \leq t \leq 8$. The tank has sufficient capacity.

(a) Find the total amount of water that enters during the first 8 minutes. Include units. [2]

(b) Find the amount of water in the tank at $t = 8$. [2]

Show sufficient work to support your answers.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 1: A storage tank

9 points

continued

Use the definitions and information on the preceding page.

(c) At $t = 4$, is the amount of water increasing or decreasing? Is its rate of change increasing or decreasing? Justify both answers. [2]

(d) Find the time at which the tank contains the greatest amount of water on $[0, 8]$, and find that amount. Justify that it is an absolute maximum. [3]

Show sufficient work to support your answers.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 2: Motion on a line

9 points

A particle moves on a line with velocity $v(t) = e^{-t/3} \sin t - 0.1$ meters per second for $0 \leq t \leq 6$. Its position at $t = 0$ is 2 meters. Use radian measure.

- (a) Find the position at $t = 6$ and the average velocity on $[0, 6]$. [2]

- (b) Find all times in $(0, 6)$ at which the particle changes direction. Explain using velocity signs. [2]

Show sufficient work to support your answers.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 2: Motion on a line**9 points**

continued

Use the definitions and information on the preceding page.

(c) Find the total distance traveled on $[0, 6]$. [2]

(d) At $t = 4$, is the speed increasing or decreasing? Show the calculus reasoning. [3]

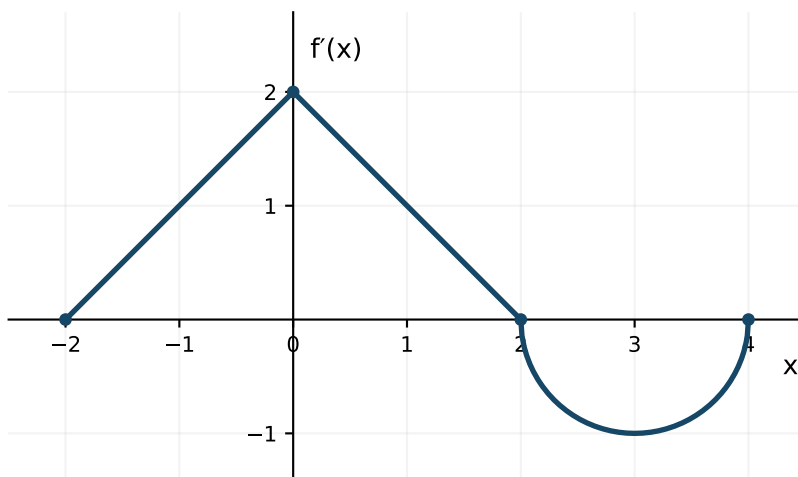
Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 3: Read a derivative graph**9 points**

The continuous graph of f' consists of line segments from $(-2, 0)$ to $(0, 2)$ to $(2, 0)$, followed by the lower semicircle of radius 1 centered at $(3, 0)$ from $x = 2$ to $x = 4$. Let $f(0) = 1$. The graph below is f' , not f .



(a) Find $f(-2)$ and $f(4)$ exactly.

[2]

(b) Find the absolute maximum and absolute minimum values of f on $[-2, 4]$. Justify.

[2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 3: Read a derivative graph**9 points**

continued

Use the definitions and information on the preceding page.

(c) Give every open interval on which f is concave up, and identify all inflection-point x -coordinates. Explain. [2]

(d) Let $h(x) = f(x^2)$. Find $h'(1)$ and $h''(1)$. [3]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 4: A separable differential equation

9 points

Let $y = f(x)$ solve $y' = (2x + 1)(y - 2)$ with $f(0) = 3$.

(a) Write the tangent line at $x = 0$, and use it to approximate $f(0.1)$. [2]

(b) Find $f''(0)$. Show the differentiation used. [2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 4: A separable differential equation**9 points**

continued

Use the definitions and information on the preceding page.

(c) Find the particular solution $f(x)$ by separation of variables.

[3]

(d) Is the tangent-line approximation in part (a) an overestimate or an underestimate? Justify on the whole interval $[0, 0.1]$, not just at 0.

[2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 5: One region, three solids

9 points

Let R be the region bounded by $y = 2x$ and $y = x^2$.

(a) Find the area of R . [2]

(b) A solid has base R and square cross sections perpendicular to the x -axis. Find its volume. [2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 5: One region, three solids

9 points

continued

Use the definitions and information on the preceding page.

(c) Write, but do not evaluate, a washer integral for the volume obtained by rotating R about $y = 4$. Identify the two radii. [2]

(d) Write and evaluate a washer integral for the volume obtained by rotating R about the y -axis. Use integration with respect to y . [3]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 6: Function values and theorems**9 points**

The function f is twice continuously differentiable. Selected values are shown, and $\int_0^3 f(x)dx = 6$.

x	0	1	2	3
$f(x)$	1	2	0	4
$f'(x)$	3	−1	2	5

(a) Let $H(x) = \int_1^{x^2} f(t)dt$. Find $H'(1)$ and $H''(1)$. [2]

(b) Explain why there is a $c \in (2, 3)$ with $f'(c) = 4$. Name the theorem and check its hypotheses. [2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 6: Function values and theorems

9 points

continued

Use the definitions and information on the preceding page.

(c) A local inverse g of f is chosen with $g(0) = 2$. Find $g'(0)$, and explain why this local inverse exists. [2]

(d) Evaluate $\int_0^3 [2f(x) + f'(x)]dx$. Show how the given integral and endpoint values enter the calculation. [3]

Show sufficient work to support your answers.