

RIGOROUS CALCULUS PRACTICE

AP Calculus BC

Worked solutions and marking guide

AB concepts plus advanced integration, polar, parametric and series

Section	Questions / time	Calculator
MCQ Part A	29 / 62 minutes	Not permitted
MCQ Part B	13 / 38 minutes	Graphing required
FRQ Part A	2 / 30 minutes	Graphing required
FRQ Part B	4 / 60 minutes	Not permitted

Total working time: **190 minutes**. MCQ and FRQ each carry 50% of this mock's weighted practice score.

How to mark fairly

Award credit for mathematically equivalent methods. The point allocations are Academy practice rubrics, not official exam-board rubrics. Give method credit for sound work even if a later arithmetic error occurs; avoid penalizing the same error twice. Do not award a reasoning point for an unsupported correct number.

Practice score: Let m be the number of correct MCQs out of 42, and r the total FRQ points out of 54. Compute $50(m/42) + 50(r/54)$ out of 100. There is no penalty for an incorrect MCQ. This score is not an AP 1–5 prediction; no official grade boundaries are supplied.

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Quick answer key

Question	Answer	Question	Answer	Question	Answer
1	B	15	C	29	C
2	B	16	B	30	C
3	D	17	C	31	A
4	A	18	B	32	C
5	D	19	D	33	D
6	B	20	C	34	A
7	C	21	B	35	C
8	D	22	D	36	A
9	A	23	A	37	C
10	D	24	B	38	C
11	A	25	B	39	B
12	D	26	D	40	C
13	C	27	A	41	B
14	C	28	D	42	C

Diagnose before you repeat

For each missed question, record the topic, the incorrect step, and the idea that fixes it. Rework the question without reading the explanation. Then return to the relevant Academy video and its lesson worksheet.

Multiple-choice worked solutions

1. Answer B

Limits

Correct choice: -2

One L'Hospital step gives $[-4x/(1+2x)]/(2x) = -2/(1+2x)$ for $x \neq 0$, so the limit is -2 .

2. Answer B

Inverse derivatives

Correct choice: $\frac{1}{2}$

Differentiate $g'(y) = 1/f'(g(y))$: $g''(y) = -f''(g(y))/[f'(g(y))]^3$. Thus $g''(3) = 4/8 = 1/2$.

3. Answer D

Implicit derivatives

Correct choice: $-\frac{7}{36}$

$2x + 3y^2y' = 0$, so $y' = -1/6$. Then $2 + 6y(y')^2 + 3y^2y'' = 0$; substituting gives $y'' = -7/36$.

4. Answer A

FTC variable limits

Correct choice: 1

$F'(x) = 2e^{-4x^2} - e^{-x^2}$ by the chain rule at both endpoints. At 0 it is $2 - 1 = 1$.

Multiple-choice worked solutions

5. Answer D

Net change

Correct choice: 3

$$\text{Average derivative} = \frac{1}{2} \int_0^2 f'(x) dx = [f(2) - f(0)]/2 = 3.$$

6. Answer B

Concavity

Correct choice: $(0, 2)$

$$f'' = (x - 2)^2 + 2(x + 1)(x - 2) = 3x(x - 2), \text{ negative exactly on } (0, 2).$$

7. Answer C

Absolute extrema

Correct choice: minimum 4, maximum 5

$$f' = 1 - 4/x^2 \text{ vanishes at } 2. \text{ Compare } f(1) = 5, f(2) = 4, \text{ and } f(4) = 5.$$

8. Answer D

Motion and speed

Correct choice: $(1, 3) \cup (5, 6)$

Speed increases when velocity and acceleration have the same sign. $v = (t - 1)(t - 5)$ and $a = 2(t - 3)$ give both negative on $(1, 3)$ and both positive on $(5, 6)$.

Multiple-choice worked solutions

9. Answer A

Substitution

Correct choice: 4

Let $u = 2x - 1$, so $du = 2dx$ and bounds become 1, 5. The value is $\frac{1}{2} \int_1^5 f(u) du = 4$.

10. Answer D

Area

Correct choice: $\frac{1}{4}$

$x \geq x^3$ on this interval, so $\text{area} = \int_0^1 (x - x^3) dx = 1/2 - 1/4 = 1/4$.

11. Answer A

Cross sections

Correct choice: $\frac{4\sqrt{3}}{3}$

Triangle side $= 2\sqrt{1 - x^2}$, so $\text{area} = (\sqrt{3}/4) \cdot 4(1 - x^2)$. Integrate from -1 to 1 to obtain $4\sqrt{3}/3$.

12. Answer D

Volumes

Correct choice: $\frac{15\pi}{2}$

Disk area $\pi(\sqrt{x})^2 = \pi x$. Thus $V = \pi[x^2/2]_1^4 = 15\pi/2$.

Multiple-choice worked solutions

13. Answer C

Separable equations

Correct choice: $y = 3e^{x^2} - 1$

Separate: $dy/(y+1) = 2x dx$. Then $y+1 = Ce^{x^2}$; the initial condition gives $C = 3$.

14. Answer C

Average value

Correct choice: $\frac{4}{\pi}$

Average = $(4/\pi)[\tan x]_0^{\pi/4} = 4/\pi$.

15. Answer C

Optimization

Correct choice: $-\frac{1}{2e}$

$f'(x) = x(2\ln x + 1)$ changes from negative to positive at $x = e^{-1/2}$. At that point $f = -1/(2e)$.

16. Answer B

Squeeze theorem

Correct choice: $\lim_{x \rightarrow 0} f(x) = 0$

Both bounding functions tend to zero, so the squeeze theorem determines the limit. The assumptions do not define $f(0)$ or impose differentiability.

Multiple-choice worked solutions

17. Answer C

Partial fractions

Correct choice: $\frac{1}{2} \ln \left| \frac{x}{x+2} \right| + C$

$1/[x(x+2)] = \frac{1}{2}(1/x - 1/(x+2))$. Integrate both terms on any interval avoiding 0, -2.

18. Answer B

Integration by parts

Correct choice: $\frac{1}{4}$

Integrate by parts with $u = \ln(1+x)$ and $dv = x dx$. Use $x^2/(1+x) = x - 1 + 1/(1+x)$. The result is $\frac{1}{2} \ln 2 - \frac{1}{2}(\ln 2 - 1/2) = 1/4$.

19. Answer D

Improper integrals

Correct choice: 1

With $u = \ln x$, the integral becomes $\int_1^\infty u^{-2} du = [-u^{-1}]_1^\infty = 1$.

20. Answer C

Parametric derivatives

Correct choice: $\frac{3}{2}$

$dy/dx = \frac{3}{2}(t - 1/t)$. Differentiate in t and divide by $dx/dt = 2t$: $d^2y/dx^2 = \frac{3(1+t^{-2})}{4t}$, yielding $3/2$.

Multiple-choice worked solutions

21. Answer B

Polar area

Correct choice: π

$A = \frac{1}{2} \int_{-\pi/2}^{\pi/2} 4 \cos^2 \theta \, d\theta = \pi$. The curve is a circle of radius 1.

22. Answer D

Vector motion

Correct choice: 1

The speed derivative is $(\mathbf{v} \cdot \mathbf{a})/|\mathbf{v}|$. At zero, $\mathbf{v} = \langle 1, 0 \rangle$ and $\mathbf{a} = \langle 1, 2 \rangle$, so it equals 1.

23. Answer A

Series divergence

Correct choice: It diverges because its terms do not approach zero.

The consecutive-term ratio is $(n+1)/3$, eventually greater than 1 and unbounded. Terms eventually grow and cannot tend to zero.

24. Answer B

Power-series endpoints

Correct choice: $[-1, 5)$

The ratio test gives $|x-2| < 3$. At $x = -1$ the alternating harmonic series converges; at $x = 5$ the harmonic series diverges.

Multiple-choice worked solutions

25. Answer B

Conditional convergence

Correct choice: conditionally convergent.

The magnitudes decrease to zero, so the alternating-series test applies. The absolute series is a divergent p -series with $p = 1/2$.

26. Answer D

Taylor coefficients

Correct choice: $-\frac{9}{2}$

$$x^2 \sin(3x) = x^2(3x - (3x)^3/3! + \cdots) = 3x^3 - (9/2)x^5 + \cdots.$$

27. Answer A

Geometric power series

Correct choice: 2

$$1/(4 + x^2) = \frac{1}{4} \sum_{n=0}^{\infty} (-x^2/4)^n. \text{ Require } x^2/4 < 1, \text{ or } |x| < 2.$$

28. Answer D

Taylor error

Correct choice: $\frac{1}{160}$

The first omitted term is $x^5/5$. At $x = 1/2$ it is $1/(32 \cdot 5) = 1/160$.

Multiple-choice worked solutions

29. Answer C

Telescoping

Correct choice: $\frac{3}{4}$

$1/[n(n+2)] = \frac{1}{2}(1/n - 1/(n+2))$. Partial sums reduce to $\frac{1}{2}(1 + 1/2 - 1/(N+1) - 1/(N+2)) \rightarrow 3/4$.

30. Answer C

Numerical root

Correct choice: 1.592

The function $\ln x + x^2 - 3$ is strictly increasing for $x > 0$, and a numerical root gives its unique zero. Numerically, the answer is 1.592143, which rounds to 1.592.

31. Answer A

Numerical volume

Correct choice: 0.364

Use disks: $V = \pi \int_0^1 [\ln(1+x^2)]^2 dx$. Numerically, the answer is 0.364390, which rounds to 0.364.

32. Answer C

Area

Correct choice: 0.389

The positive intersection is $a \approx 0.703467422$. Integrate upper minus lower: $\int_0^a (e^{-x} - x^2) dx$. Numerically, the answer is 0.389093, which rounds to 0.389.

Multiple-choice worked solutions

33. Answer D

Absolute extrema

Correct choice: 0.503

Solve $f'(x) = e^{-x^2}(1 - 2x^2) + 0.1 = 0$. Critical numbers are approximately 0.768257 and 2.083658. Compare function values there and at both endpoints. Numerically, the answer is 0.502598, which rounds to 0.503.

34. Answer A

Average derivative

Correct choice: -0.215

By FTC, the average is $[f(3) - f(1)]/(3 - 1) = (\sin 9 - \sin 1)/2$. Use radian mode. Numerically, the answer is -0.214676 , which rounds to -0.215 .

35. Answer C

Inverse derivative

Correct choice: 0.149

Solve $f(a) = 5$, giving $a \approx 1.378796700$. The inverse derivative is $g'(5) = 1/f'(a) = 1/(3a^2 + 1)$. Numerically, the answer is 0.149182, which rounds to 0.149.

36. Answer A

Tabular integration

Correct choice: 28.500

Square the function values first. Then $1(1 + 4)/2 + 1(4 + 9)/2 + 3(9 + 4)/2 = 28.5$. Numerically, the answer is 28.500000, which rounds to 28.500.

Multiple-choice worked solutions

37. Answer C

Total distance

Correct choice: 4.080

Break at $\arcsin(0.2)$ and $\pi - \arcsin(0.2)$. Integrate the absolute value of velocity across the three sign intervals. Numerically, the answer is 4.080270, which rounds to 4.080.

38. Answer C

Euler method

Correct choice: 1.856

Use successive left-end slopes: $y_1 = 1 + 0.2(1) = 1.2$; $y_2 = 1.2 + 0.2(0.2 + 1.2) = 1.48$; $y_3 = 1.48 + 0.2(0.4 + 1.48) = 1.856$. Numerically, the answer is 1.856000, which rounds to 1.856.

39. Answer B

Parametric arc length

Correct choice: 4.335

Length $= \int_0^2 \sqrt{(2t)^2 + [1/(1+t)]^2} dt$. Integrate numerically. Numerically, the answer is 4.335285, which rounds to 4.335.

40. Answer C

Polar intersection area

Correct choice: 1.010

The crossing angle is $\alpha \approx 0.927295218$. The smaller radius is $2 \sin \theta$ before α and $1 + \cos \theta$ after it. Use $\frac{1}{2} \int_0^\alpha (2 \sin \theta)^2 d\theta + \frac{1}{2} \int_\alpha^{\pi/2} (1 + \cos \theta)^2 d\theta$. Numerically, the answer is 1.009921, which rounds to 1.010.

Multiple-choice worked solutions

41. Answer B

Logistic growth

Correct choice: 7.324

The solution is $P(t) = 500/(1 + 9e^{-0.3t})$. Setting $P = 250$ gives $9e^{-0.3t} = 1$, hence $t = \ln 9/0.3$. Numerically, the answer is 7.324082, which rounds to 7.324.

42. Answer C

Lagrange remainder

Correct choice: 0.038

$f^{(4)}(x) = 16e^{2x}$ has maximum $16e^{0.8}$. Thus $|R_3(0.4)| \leq 16e^{0.8}(0.4)^4/4! = e^{0.8}(0.8)^4/24$. Numerically, the answer is 0.037983, which rounds to 0.038.

Free-response solution 1

Rates measured at uneven times

Tables, rates, IVT, limits of inference

9 points

(a) [2] $\int_0^8 r(t)dt \approx 2(4+7)/2 + 3(7+5)/2 + 3(5+1)/2 = 38$ cubic meters.

Marking: 1 point for trapezoids with actual widths; 1 for the estimate.

(b) [2] $\int_0^8 s(t)dt = [2t + t^2/8]_0^8 = 24$. Thus $B(8) \approx 40 + 38 - 24 = 54$ cubic meters.

Marking: 1 point for total outflow; 1 for initial volume plus net change.

(c) [2] $r'(5) \approx [r(8) - r(2)]/(8 - 2) = -1$. Since $B'' = r' - s'$, $B''(5) \approx -1 - 0.25 = -1.25$ cubic meters per hour squared.

Marking: 1 point for the secant estimate; 1 for the second derivative with units.

(d) [3] $B' = r - s$ is continuous; $B'(5) = 5 - 3.25 = 1.75 > 0$ and $B'(8) = 1 - 4 = -3 < 0$. The Intermediate Value Theorem guarantees a zero between them. Values at four times do not control all intervening rate changes, so they cannot establish a unique absolute maximum.

Marking: 1 point for continuity; 1 for endpoint signs and IVT; 1 for the limitation.

Equivalent sound methods earn the same credit.

Free-response solution 2

A planar trajectory

Vector motion and parametric curves

9 points

(a) [2] $x(3) = 1 + \int_0^3 [1 + \cos(t^2)] dt \approx 4.703$ and $y(3) = -2 + \int_0^3 [\sin t + 0.3] dt \approx 0.890$.

Marking: 1 point for each coordinate with initial position and integral.

(b) [2] Speed is $\sqrt{(1 + \cos 4)^2 + (\sin 2 + 0.3)^2} \approx 1.258$ m/s. Acceleration is $\langle -2t \sin(t^2), \cos t \rangle$, so $\mathbf{a}(2) \approx \langle 3.027, -0.416 \rangle$ m/s².

Marking: 1 point for speed; 1 for the acceleration vector.

(c) [2] $D = \int_0^3 \sqrt{[1 + \cos(t^2)]^2 + [\sin t + 0.3]^2} dt \approx 5.082$ meters.

Marking: 1 point for the speed integral; 1 for the distance.

(d) [3] A vertical tangent requires $x' = 0$ and $y' \neq 0$. Here $t^2 = (2k + 1)\pi$; only $t = \sqrt{\pi} \approx 1.772$ lies in $(0, 3)$, since $\sqrt{3\pi} > 3$. At that time $y' = \sin \sqrt{\pi} + 0.3 > 0$.

Marking: 1 point for the condition; 1 for the unique time; 1 for nonzero vertical velocity.

Equivalent sound methods earn the same credit.

Free-response solution 3

An accumulation function

Accumulation, graph and chain rule

9 points

(a) [2] On $[-1, 1]$, the equal triangles cancel, so $A(1) = 0$. They also cancel on $[1, 3]$. From 3 to 4 the negative triangle has area -1 , so $A(4) = -1$.

Marking: 1 point for each value with signed-area reasoning.

(b) [2] $A' = g$ has signs $-, +, -, +$ separated by zeros $0, 2, 4$. Thus there are local minima $(0, -1)$ and $(4, -1)$, and a local maximum $(2, 1)$.

Marking: 1 point for the three extrema and values; 1 for the sign changes.

(c) [2] $A'' = g' < 0$ on $(1, 3)$ only. Concavity changes at $x = 1, 3$, where the slope of g switches sign.

Marking: 1 point for the interval; 1 for both changes with reasoning.

(d) [3] $J' = e^{A^2}g$ and $J'' = e^{A^2}(2Ag^2 + g')$. At 2, $A = 1$, $g = 0$, and $g' = -2$, so $J'(2) = 0$ and $J''(2) = -2e$.

Marking: 1 point for the first derivative and value; 1 for the second derivative formula; 1 for its value.

Equivalent sound methods earn the same credit.

Free-response solution 4

A differential equation and its interval

Euler method, separable equation, solution interval

9 points

(a) [2] $y_1 = 2 + 0.25(0)(2^2) = 2$ at $x = 0.25$. Then $y_2 = 2 + 0.25(0.25)(2^2) = 2.25$ at $x = 0.5$.

Marking: 1 point for each correctly displayed update.

(b) [2] $y^{-2}dy = x dx$ gives $-1/y = x^2/2 + C$. From $f(0) = 2$, $C = -1/2$, so $f(x) = 2/(1 - x^2)$.

Marking: 1 point for separation and integration; 1 for the initial-condition solution.

(c) [2] The denominator vanishes at $x = \pm 1$. Between those points the formula solves the equation smoothly. The largest such interval containing 0 is $(-1, 1)$.

Marking: 1 point for the singular points; 1 for the correct interval.

(d) [3] $y'' = y^2 + 2xyy' = y^2 + 2x^2y^3 > 0$ on $[0, 0.5]$ since $y > 0$. Successive forward tangent steps lie below this increasing, concave-up solution (the slope xy^2 also increases with positive y). Directly, $f(0.5) = 8/3 > 2.25$.

Marking: 1 point for the second derivative; 1 for positive concavity on the interval; 1 for the justified underestimate, with exact comparison also acceptable.

Equivalent sound methods earn the same credit.

Free-response solution 5

Polar geometry

Polar slope, area and length

9 points

(a) [2] $x' = r' \cos \theta - r \sin \theta$ and $y' = r' \sin \theta + r \cos \theta$, where $r' = -\sin \theta$. At $\pi/2$, $x' = -2$ and $y' = -1$, so $dy/dx = 1/2$.

Marking: 1 point for both derivative formulas; 1 for the slope.

(b) [2] $A = \frac{1}{2} \int_0^{2\pi} (2 + \cos \theta)^2 d\theta = \frac{1}{2}(8\pi + 0 + \pi) = 9\pi/2$.

Marking: 1 point for the polar area integral; 1 for its value.

(c) [3] The curve is outside $r = 2$ when $\cos \theta \geq 0$. Using $-\pi/2 \leq \theta \leq \pi/2$, $A = \frac{1}{2} \int_{-\pi/2}^{\pi/2} [(2 + \cos \theta)^2 - 4] d\theta = 4 + \pi/4$.

Marking: 1 point for the interval; 1 for subtracting squared radii; 1 for the exact area.

(d) [2] $L = \int_0^{2\pi} \sqrt{r^2 + (r')^2} d\theta = \int_0^{2\pi} \sqrt{5 + 4 \cos \theta} d\theta$.

Marking: 1 point for the polar arc-length formula; 1 for the correct integrand and bounds.

Equivalent sound methods earn the same credit.

Free-response solution 6

Series, endpoints, and an error bound

Power series and justified error bounds

9 points

(a) [2] Substitute $u = t^2$ and integrate: $F(x) = x^3/3 - x^5/10 + x^7/21 - \dots = \sum_{n=1}^{\infty} (-1)^{n+1} x^{2n+1} / [n(2n+1)]$.

Marking: 1 point for the three terms; 1 for the general term.

(b) [2] The radius is 1 by the ratio test. At each endpoint the absolute terms are $1/[n(2n+1)] \leq 1/(2n^2)$, so both endpoints converge absolutely by comparison. The interval is $[-1, 1]$.

Marking: 1 point for radius/interior; 1 for both endpoint tests and the interval.

(c) [2] $F(1/2) \approx (1/2)^3/3 - (1/2)^5/10 = 1/24 - 1/320 = 37/960$.

Marking: 1 point for substitution; 1 for the fraction.

(d) [3] The first omitted term is $(1/2)^7/21 = 1/2688$. Terms alternate and their magnitudes decrease to zero, so $|R| \leq 1/2688$. The omitted term is positive, so the approximation is below the true value.

Marking: 1 point for the bound; 1 for the alternating-series conditions; 1 for the direction of error.

Equivalent sound methods earn the same credit.