

RIGOROUS CALCULUS PRACTICE

AP Calculus BC

Full-length mock | 2027 structure

AB concepts plus advanced integration, polar, parametric and series

Section	Questions / time	Calculator
MCQ Part A	29 / 62 minutes	Not permitted
MCQ Part B	13 / 38 minutes	Graphing required
FRQ Part A	2 / 30 minutes	Graphing required
FRQ Part B	4 / 60 minutes	Not permitted

Total working time: **190 minutes**. MCQ and FRQ each carry 50% of this mock's weighted practice score.

Before you begin

Write your name and date below. Keep the separate solutions file closed until you finish. Show equations, reasoning, units and required justifications. Use radian mode. Unless an exact answer is requested, round final numerical answers to three decimal places and retain unrounded values during calculations.

Choose exactly one answer for each MCQ. Use the answer sheet at the back of Section I. Time the four parts separately and keep the calculator closed in the two no-calculator parts. These printable pages do not simulate the Bluebook interface.

Name: _____ Date: _____

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Section I | Part A**No calculator**

29 questions / 62 minutes

1. Evaluate $\lim_{x \rightarrow 0} \frac{\ln(1 + 2x) - 2x}{x^2}$.

- (A) 0
- (B) -2
- (C) 2
- (D) -1

2. A one-to-one function has $f(1) = 3$, $f'(1) = 2$, and $f''(1) = -4$. For $g = f^{-1}$, find $g''(3)$.

- (A) $-\frac{1}{2}$
- (B) $\frac{1}{2}$
- (C) -2
- (D) 2

3. For $x^2 + y^3 = 9$, find y'' at $(1, 2)$.

- (A) $-\frac{1}{6}$
- (B) $-\frac{7}{18}$
- (C) $\frac{7}{36}$
- (D) $-\frac{7}{36}$

Section I | Part A**No calculator**

29 questions / 62 minutes

4. If $F(x) = \int_x^{2x} e^{-t^2} dt$, find $F'(0)$.

- (A) 1
- (B) 0
- (C) 2
- (D) -1

5. A differentiable function has continuous derivative, $f(0) = 1$, and $f(2) = 7$. What is the average value of f' on $[0, 2]$?

- (A) 6
- (B) $\frac{7}{2}$
- (C) 4
- (D) 3

6. If $f'(x) = (x + 1)(x - 2)^2$, where is f concave down?

- (A) $(-\infty, 0)$
- (B) $(0, 2)$
- (C) $(2, \infty)$
- (D) $(-1, 2)$

Section I | Part A**No calculator**

29 questions / 62 minutes

7. What are the absolute minimum and maximum values of $f(x) = x + 4/x$ on $[1, 4]$?

- (A) minimum 4, maximum 8
- (B) minimum 2, maximum 5
- (C) minimum 4, maximum 5
- (D) minimum 5, maximum 8

8. A particle has velocity $v(t) = t^2 - 6t + 5$ for $0 < t < 6$. On which intervals is its speed increasing?

- (A) $(0, 1) \cup (3, 5)$
- (B) $(3, 6)$
- (C) $(0, 3)$
- (D) $(1, 3) \cup (5, 6)$

9. If $\int_1^5 f(u)du = 8$, find $\int_1^3 f(2x - 1)dx$.

- (A) 4
- (B) 8
- (C) 16
- (D) 2

Section I | Part A**No calculator**

29 questions / 62 minutes

10. Find the area between $y = x$ and $y = x^3$ for $0 \leq x \leq 1$.

- (A) $\frac{1}{2}$
- (B) $\frac{1}{12}$
- (C) $\frac{3}{4}$
- (D) $\frac{1}{4}$

11. A solid has base the disk $x^2 + y^2 \leq 1$. Its cross sections perpendicular to the x -axis are equilateral triangles with bases in the disk. Find its volume.

- (A) $\frac{4\sqrt{3}}{3}$
- (B) $\sqrt{3}$
- (C) $\frac{2\sqrt{3}}{3}$
- (D) π

12. The region below $y = \sqrt{x}$, above $y = 0$, and between $x = 1$ and $x = 4$ is rotated about the x -axis. Find the volume.

- (A) 15π
- (B) 3π
- (C) 7π
- (D) $\frac{15\pi}{2}$

Section I | Part A**No calculator**

29 questions / 62 minutes

13. Solve $y' = 2x(y + 1)$ with $y(0) = 2$.

- (A) $y = 2e^{x^2}$
(B) $y = e^{x^2} + 1$
(C) $y = 3e^{x^2} - 1$
(D) $y = 3e^{2x} - 1$

14. Find the average value of $\sec^2 x$ on $[0, \pi/4]$.

- (A) 1
(B) $\frac{\pi}{4}$
(C) $\frac{4}{\pi}$
(D) $\frac{2}{\pi}$

15. For $x > 0$, what is the minimum value of $f(x) = x^2 \ln x$?

- (A) 0
(B) $-\frac{1}{e}$
(C) $-\frac{1}{2e}$
(D) $-\frac{1}{e^2}$

Section I | Part A**No calculator**

29 questions / 62 minutes

16. For $x \neq 0$, suppose $-|x| \leq f(x) \leq |x|$. What must be true?

- (A) $f'(0) = 0$
- (B) $\lim_{x \rightarrow 0} f(x) = 0$
- (C) f is differentiable near 0.
- (D) $f(0) = 0$

17. An antiderivative of $1/[x(x+2)]$ is

- (A) $\ln|x| - \ln|x+2| + C$
- (B) $\frac{1}{2} \ln|x+2| + C$
- (C) $\frac{1}{2} \ln \left| \frac{x}{x+2} \right| + C$
- (D) $\ln|x(x+2)| + C$

18. Evaluate $\int_0^1 x \ln(1+x) dx$.

- (A) $\frac{1}{2}$
- (B) $\frac{1}{4}$
- (C) $\frac{\ln 2}{2}$
- (D) $\ln 2 - \frac{1}{2}$

Section I | Part A**No calculator**

29 questions / 62 minutes

19. Evaluate $\int_e^\infty \frac{dx}{x(\ln x)^2}$.

- (A) 0
- (B) e^{-1}
- (C) The integral diverges.
- (D) 1

20. A curve has $x = t^2 + 1$, $y = t^3 - 3t$. Find d^2y/dx^2 at $t = 1$.

- (A) 6
- (B) 0
- (C) $\frac{3}{2}$
- (D) 3

21. Find the area enclosed by the polar curve $r = 2 \cos \theta$, traced once for $-\pi/2 \leq \theta \leq \pi/2$.

- (A) 2π
- (B) π
- (C) 4π
- (D) $\frac{\pi}{2}$

Section I | Part A**No calculator**

29 questions / 62 minutes

22. A particle has velocity $\mathbf{v}(t) = \langle e^t, 2t \rangle$. Find the rate of change of its speed at $t = 0$.

- (A) 2
- (B) 0
- (C) $\sqrt{5}$
- (D) 1

23. Which statement describes $\sum_{n=1}^{\infty} \frac{n!}{3^n}$?

- (A) It diverges because its terms do not approach zero.
- (B) It converges conditionally.
- (C) It converges by the ratio test.
- (D) It is a convergent geometric series.

24. Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$.

- (A) $[-1, 5]$
- (B) $[-1, 5)$
- (C) $(-1, 5)$
- (D) $(-1, 5]$

Section I | Part A**No calculator**

29 questions / 62 minutes

25. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ is

- (A) divergent because $1/\sqrt{n}$ diverges.
- (B) conditionally convergent.
- (C) divergent because the terms fail to approach zero.
- (D) absolutely convergent.

26. Find the coefficient of x^5 in the Maclaurin series for $x^2 \sin(3x)$.

- (A) $-\frac{27}{120}$
- (B) $\frac{9}{2}$
- (C) 3
- (D) $-\frac{9}{2}$

27. What is the radius of convergence of the Maclaurin series for $1/(4 + x^2)$?

- (A) 2
- (B) ∞
- (C) $\frac{1}{2}$
- (D) 4

Section I | Part A**No calculator**

29 questions / 62 minutes

28. Use $x - x^3/3$ to approximate $\arctan x$ at $x = 1/2$. What alternating-series bound follows for the absolute error?

- (A) $\frac{1}{448}$
- (B) $\frac{1}{32}$
- (C) $\frac{1}{24}$
- (D) $\frac{1}{160}$

29. Evaluate $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$.

- (A) 1
- (B) $\frac{1}{2}$
- (C) $\frac{3}{4}$
- (D) $\frac{3}{2}$

STOP. End of this part. Check your responses before moving to the next timed part.

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

30. Find the positive solution of $\ln x + x^2 = 3$.

- (A) 1.842
- (B) 1.732
- (C) 1.592
- (D) 1.342

31. The region below $y = \ln(1 + x^2)$ on $0 \leq x \leq 1$ is rotated about the x -axis. Find the volume.

- (A) 0.364
- (B) 0.729
- (C) 0.693
- (D) 0.116

32. Find the area enclosed by $y = e^{-x}$, $y = x^2$, and the y -axis in the first quadrant.

- (A) 0.539
- (B) 0.611
- (C) 0.389
- (D) 0.195

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

33. Find the absolute maximum value of $f(x) = xe^{-x^2} + 0.1x$ on $[0, 3]$.

- (A) 0.300
- (B) 0.768
- (C) 0.235
- (D) 0.503

34. For $f(x) = \sin(x^2)$, find the average value of f' on $[1, 3]$.

- (A) -0.215
- (B) 0.215
- (C) 0.107
- (D) -0.429

35. Let $f(x) = x^3 + x + 1$ and $g = f^{-1}$. Find $g'(5)$.

- (A) 0.448
- (B) 6.703
- (C) 0.149
- (D) 1.379

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

36. The table gives $f(x)$. Estimate $\int_0^5 [f(x)]^2 dx$ with trapezoids on the three listed subintervals.

x	0	1	2	5
$f(x)$	1	2	3	2

- (A) 28.500
- (B) 24.500
- (C) 32.500
- (D) 12.000

37. A particle has velocity $v(t) = \sin t - 0.2$ on $[0, 2\pi]$. Find total distance traveled.

- (A) 1.757
- (B) 4.000
- (C) 4.080
- (D) 1.257

38. Use Euler's method with step $h = 0.2$ for $y' = x + y$, $y(0) = 1$, to estimate $y(0.6)$.

- (A) 1.728
- (B) 1.480
- (C) 1.856
- (D) 1.822

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

39. Find the arc length of $x = t^2$, $y = \ln(1 + t)$ for $0 \leq t \leq 2$.

- (A) 4.148
- (B) 4.335
- (C) 2.168
- (D) 5.335

40. In the first quadrant, find the area inside both $r = 1 + \cos \theta$ and $r = 2 \sin \theta$.

- (A) 1.571
- (B) 0.505
- (C) 1.010
- (D) 2.020

41. A population satisfies $P' = 0.3P(1 - P/500)$ and $P(0) = 50$. When does it first reach 250?

- (A) 5.365
- (B) 7.324
- (C) 3.662
- (D) 2.197

Section I | Part B**Graphing calculator required**

13 questions / 38 minutes

42. Use the degree-3 Maclaurin polynomial for e^{2x} at $x = 0.4$. What Lagrange error bound follows using the maximum fourth derivative on $[0, 0.4]$?

- (A) 0.608
- (B) 0.017
- (C) 0.038
- (D) 0.002

STOP. End of this part. Check your responses before moving to the next timed part.

Multiple-choice answer sheet

Mark one response per question.

- | | | |
|---|---|---|
| 1. ☐ A ☐ B ☐ C ☐ D | 15. ☐ A ☐ B ☐ C ☐ D | 29. ☐ A ☐ B ☐ C ☐ D |
| 2. ☐ A ☐ B ☐ C ☐ D | 16. ☐ A ☐ B ☐ C ☐ D | 30. ☐ A ☐ B ☐ C ☐ D |
| 3. ☐ A ☐ B ☐ C ☐ D | 17. ☐ A ☐ B ☐ C ☐ D | 31. ☐ A ☐ B ☐ C ☐ D |
| 4. ☐ A ☐ B ☐ C ☐ D | 18. ☐ A ☐ B ☐ C ☐ D | 32. ☐ A ☐ B ☐ C ☐ D |
| 5. ☐ A ☐ B ☐ C ☐ D | 19. ☐ A ☐ B ☐ C ☐ D | 33. ☐ A ☐ B ☐ C ☐ D |
| 6. ☐ A ☐ B ☐ C ☐ D | 20. ☐ A ☐ B ☐ C ☐ D | 34. ☐ A ☐ B ☐ C ☐ D |
| 7. ☐ A ☐ B ☐ C ☐ D | 21. ☐ A ☐ B ☐ C ☐ D | 35. ☐ A ☐ B ☐ C ☐ D |
| 8. ☐ A ☐ B ☐ C ☐ D | 22. ☐ A ☐ B ☐ C ☐ D | 36. ☐ A ☐ B ☐ C ☐ D |
| 9. ☐ A ☐ B ☐ C ☐ D | 23. ☐ A ☐ B ☐ C ☐ D | 37. ☐ A ☐ B ☐ C ☐ D |
| 10. ☐ A ☐ B ☐ C ☐ D | 24. ☐ A ☐ B ☐ C ☐ D | 38. ☐ A ☐ B ☐ C ☐ D |
| 11. ☐ A ☐ B ☐ C ☐ D | 25. ☐ A ☐ B ☐ C ☐ D | 39. ☐ A ☐ B ☐ C ☐ D |
| 12. ☐ A ☐ B ☐ C ☐ D | 26. ☐ A ☐ B ☐ C ☐ D | 40. ☐ A ☐ B ☐ C ☐ D |
| 13. ☐ A ☐ B ☐ C ☐ D | 27. ☐ A ☐ B ☐ C ☐ D | 41. ☐ A ☐ B ☐ C ☐ D |
| 14. ☐ A ☐ B ☐ C ☐ D | 28. ☐ A ☐ B ☐ C ☐ D | 42. ☐ A ☐ B ☐ C ☐ D |

End of Section I.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 1: Rates measured at uneven times

9 points

A reservoir contains 40 cubic meters at $t = 0$. Its inflow $r(t)$ is twice continuously differentiable, with values in the table. Outflow is $s(t) = 2 + 0.25t$ cubic meters per hour. Let $B(t)$ be the water volume.

t (hours)	0	2	5	8
$r(t)$ (m ³ /hour)	4	7	5	1

- (a) Use trapezoids on the three listed subintervals to estimate the total inflow on $[0, 8]$. [2]

- (b) Use your estimate to approximate $B(8)$. [2]

Show sufficient work to support your answers.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 1: Rates measured at uneven times

9 points

continued

Use the definitions and information on the preceding page.

(c) Use the secant slope of r on $[2, 8]$ to estimate $r'(5)$. Then estimate $B''(5)$ and give its units. [2]

(d) Prove that $B'(c) = 0$ for at least one $c \in (5, 8)$. Do the data establish a unique absolute maximum of B on $[0, 8]$? Explain. [3]

Show sufficient work to support your answers.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 2: A planar trajectory

9 points

A particle starts at $(1, -2)$ at $t = 0$ and has velocity components $x'(t) = 1 + \cos(t^2)$ and $y'(t) = \sin t + 0.3$ for $0 \leq t \leq 3$. Position is measured in meters and time in seconds.

(a) Find the particle's position at $t = 3$. [2]

(b) Find the speed and acceleration vector at $t = 2$. [2]

Show sufficient work to support your answers.

Section II | Part A

Graphing calculator required | 30 minutes for Questions 1–2

Question 2: A planar trajectory

9 points

continued

Use the definitions and information on the preceding page.

(c) Find the total distance traveled on $[0, 3]$.

[2]

(d) Find all times in $(0, 3)$ when the trajectory has a vertical tangent. Justify that the tangent is vertical at each listed time.

[3]

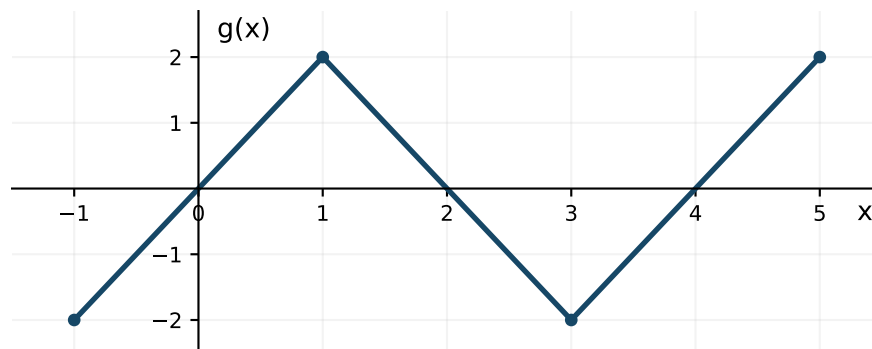
Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 3: An accumulation function**9 points**

A continuous function g is defined on $[-1, 5]$ by the line segments joining $(-1, -2)$, $(1, 2)$, $(3, -2)$, and $(5, 2)$. Let $A(x) = \int_{-1}^x g(t) dt$.



(a) Find $A(1)$ and $A(4)$. Use signed areas.

[2]

(b) Find every local maximum and local minimum of A in $(-1, 5)$. Give each coordinate $(x, A(x))$ and justify.

[2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 3: An accumulation function

9 points

continued

Use the definitions and information on the preceding page.

(c) On what intervals is A concave down? Where does its concavity change? [2]

(d) Let $J(x) = \int_0^{A(x)} e^{u^2} du$. Find $J'(2)$ and $J''(2)$. [3]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 4: A differential equation and its interval

9 points

Let $y = f(x)$ solve $y' = xy^2$ with $f(0) = 2$.

(a) Use Euler's method with two steps of size 0.25 to estimate $f(0.5)$. Show both updates. [2]

(b) Find the particular solution by separation of variables. [2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 4: A differential equation and its interval

9 points

continued

Use the definitions and information on the preceding page.

(c) Find the largest open interval containing 0 on which this solution is finite and differentiable. [2]

(d) Determine whether the Euler estimate in part (a) is below or above the exact value. Justify using the concavity of the solution on $[0, 0.5]$. [3]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 5: Polar geometry

9 points

Consider the polar curve $r = 2 + \cos \theta$ for $0 \leq \theta \leq 2\pi$. It is traced once and r is positive throughout.

(a) Find dy/dx at $\theta = \pi/2$. Show the parametric derivatives. [2]

(b) Find the total area enclosed by the curve. [2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 5: Polar geometry**9 points**

continued

Use the definitions and information on the preceding page.

(c) Find the area inside this curve and outside the circle $r = 2$.

[3]

(d) Write, but do not evaluate, an integral for the total arc length of the polar curve.

[2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 6: Series, endpoints, and an error bound

9 points

Let $F(x) = \int_0^x \ln(1+t^2)dt$. You may use $\ln(1+u) = \sum_{n=1}^{\infty} (-1)^{n+1} u^n/n$ for $-1 < u \leq 1$.

(a) Write the first three nonzero terms and the general term of the Maclaurin series for F . [2]

(b) Find the interval of convergence of this series. Justify what happens at both endpoints. [2]

Show sufficient work to support your answers.

Section II | Part B

No calculator | 60 minutes for Questions 3–6

Question 6: Series, endpoints, and an error bound

9 points

continued

Use the definitions and information on the preceding page.

(c) Use the first two nonzero terms to approximate $F(1/2)$. Give an exact fraction for the approximation.
[2]

(d) Give an alternating-series upper bound for the absolute error in part (c). Is that approximation above or below the true value? Justify. [3]

Show sufficient work to support your answers.