

RIGOROUS CALCULUS PRACTICE

China-inspired

Calculus challenge mock

Parameter cases, root counting and inequality proofs

60 minutes | 3 structured questions | 30 marks

No calculator. Give exact answers and complete mathematical justifications.

This is a calculus-focused enrichment paper inspired by selected exam approaches. It is not a full national examination or a claim of syllabus equivalence. Europe has many different examination systems.

Before you begin

Write your name and date below. Keep the separate solutions file closed until you finish. Show equations, reasoning, units and required justifications. Use radian mode. Unless an exact answer is requested, round final numerical answers to three decimal places and retain unrounded values during calculations.

Name: _____ Date: _____

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Structured challenge

No calculator | 60 minutes for all three questions

Question 1: A logarithm with a parameter

10 marks

For $a > 0$, define $f_a(x) = \ln x - a(x - 1)$ on $x > 0$. Give exact answers and justify all parameter cases.

(a) Find the intervals of increase and decrease and the maximum value of f_a . [3]

(b) Prove that $f_a(x) \leq 0$ for every $x > 0$ if and only if $a = 1$. [3]

Show sufficient work to support your answers.

Structured challenge

No calculator | 60 minutes for all three questions

Question 1: A logarithm with a parameter

10 marks

continued

Use the definitions and information on the preceding page.

(c) For fixed $a > 0$, determine the number of positive solutions of $\ln x - ax = b$ for every real b . [2]

(d) When $a = 1$, show that $f_1(x) = 0$ has exactly one solution, and explain why this zero is not a crossing of the x -axis. [2]

Show sufficient work to support your answers.

Structured challenge

No calculator | 60 minutes for all three questions

Question 2: An absolute maximum for every parameter

10 marks

Let $g_a(x) = x^3 - 3ax$ on $[-1, 1]$, where a is any real number. Define $M(a) = \max_{-1 \leq x \leq 1} g_a(x)$.

(a) Determine $M(a)$ when $a \leq 0$ and when $a \geq 1$. Include where it is attained. [2]

(b) For $0 < a < 1$, find all interior stationary points and classify them. List every candidate value for the absolute maximum. [3]

Show sufficient work to support your answers.

Structured challenge

No calculator | 60 minutes for all three questions

Question 2: An absolute maximum for every parameter

10 marks

continued

Use the definitions and information on the preceding page.

(c) Prove that the local maximum value is at least $g_a(-1)$, and find when it equals $g_a(1)$. [3]

(d) Give a complete piecewise formula for $M(a)$ for all real a , identifying the maximizing point or points in every case. [2]

Show sufficient work to support your answers.

Structured challenge

No calculator | 60 minutes for all three questions

Question 3: The sharp constant in an inequality

10 marks

For $x > 0$, define $q(x) = [x - \ln(1 + x)]/x^2$. The word *sharp* means the largest constant for which an inequality holds throughout the stated interval.

(a) Prove that $\ln(1 + x) < x$ for every $x > 0$. [2]

(b) Show that $q'(x) < 0$ for $x > 0$. You may analyze the derivative of $H(x) = x^2/(1 + x) - 2x + 2\ln(1 + x)$. [3]

Show sufficient work to support your answers.

Structured challenge

No calculator | 60 minutes for all three questions

Question 3: The sharp constant in an inequality

10 marks

continued

Use the definitions and information on the preceding page.

(c) Find the sharp constant C such that $\ln(1+x) \leq x - Cx^2$ for all $0 < x \leq 1$. Prove both validity and sharpness. [3]

(d) Find $\lim_{x \rightarrow 0^+} q(x)$ and hence state strict lower and upper bounds for $q(x)$ on $0 < x < 1$. [2]

Show sufficient work to support your answers.