

RIGOROUS CALCULUS PRACTICE

European approaches

Worked solutions and marking guide

Selected A-level and French-style themes: models, sequences and proof

60 minutes | 3 structured questions | 30 marks

A graphing calculator may be used for numerical work. Exact answers and proofs are still required where requested.

This is a calculus-focused enrichment paper inspired by selected exam approaches. It is not a full national examination or a claim of syllabus equivalence. Europe has many different examination systems.

How to mark fairly

Award credit for mathematically equivalent methods. The point allocations are Academy practice rubrics, not official exam-board rubrics. Give method credit for sound work even if a later arithmetic error occurs; avoid penalizing the same error twice. Do not award a reasoning point for an unsupported correct number.

Add the marks across the three questions for a score out of 30. Use the topic labels to plan revision. No national grade conversion is implied.

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Structured solution 1

Exponential growth with a carrying capacity

Logistic equation and model evaluation

10 marks

(a) [2] Equilibria are $P = 0$ and $P = 800$. Starting at 100, the solution remains between them and increases because $P' > 0$ there; it approaches 800.

Marking: 1 mark for equilibria; 1 for the increasing solution and limiting population.

(b) [3] $800 \, dP/[P(800 - P)] = 0.2 \, dt$ and $800/[P(800 - P)] = 1/P + 1/(800 - P)$. Integration gives $\ln(P/(800 - P)) = 0.2t + C$. Initially $P/(800 - P) = 1/7$, which rearranges to the stated solution.

Marking: 1 mark for separation and fractions; 1 for integration; 1 for applying the initial condition and solving.

(c) [3] As a function of P , the rate $0.2P - P^2/4000$ is a concave quadratic with maximum at $P = 400$. Its value is 40 individuals/year. From $1 + 7e^{-0.2t} = 2$, the time is $5 \ln 7 \approx 9.730$ years.

Marking: 1 mark for maximizing at 400; 1 for the rate; 1 for the time.

(d) [2] $600 = 800/(1 + 7e^{-0.2t})$ yields $e^{-0.2t} = 1/21$, so $t = 5 \ln 21 \approx 15.223$ years. A changing habitat, migration, or a changing growth parameter could invalidate the fixed-capacity model.

Marking: 1 mark for time; 1 for a relevant model limitation.

Equivalent sound methods earn the same credit.

Structured solution 2

Newton iteration and proof of convergence

Numerical methods, induction, convergence

10 marks

(a) [2] Newton gives $u_{n+1} = u_n - (u_n^2 - 2)/(2u_n) = \frac{1}{2}(u_n + 2/u_n)$. Hence $u_1 = 3/2$ and $u_2 = 17/12$.

Marking: 1 mark for the Newton derivation; 1 for both exact iterates.

(b) [3] $u_{n+1} - \sqrt{2} = (u_n^2 - 2\sqrt{2}u_n + 2)/(2u_n) = (u_n - \sqrt{2})^2/(2u_n)$. The base value $2 > \sqrt{2}$ is positive, and if $u_n > \sqrt{2}$, the identity is strictly positive. Induction proves the claim.

Marking: 1 mark for the identity; 1 for the base case; 1 for the induction step.

(c) [3] $u_{n+1} - u_n = (2 - u_n^2)/(2u_n) < 0$. It is decreasing and bounded below, hence converges to some $L \geq \sqrt{2}$. Taking limits in the recurrence yields $L = (L + 2/L)/2$, so $L^2 = 2$ and $L = \sqrt{2}$.

Marking: 1 mark for decrease; 1 for bounded convergence; 1 for identifying the limit.

(d) [2] Using $u_n > \sqrt{2}$ in the identity from (b) gives the stated upper bound. Once the error is small, the next error is bounded by a constant times its square: quadratic convergence.

Marking: 1 mark for the inequality; 1 for interpreting the squared error.

Equivalent sound methods earn the same credit.

Structured solution 3

A response curve and its parameters

Exponential modeling, improper integral, sensitivity

10 marks

(a) [2] $C' = Ae^{-kt}(1 - kt)$ changes from positive to negative at $t = 1/k$. The peak value is $A/(ke)$.

Marking: 1 mark for the derivative and signs; 1 for both peak expressions.

(b) [2] $1/k = 2$ gives $k = 1/2$. Then $A/(ke) = 12$ gives $A = 6e$.

Marking: 1 mark for each parameter.

(c) [4] Integration by parts gives $\int_0^T Ate^{-kt}dt = \frac{A}{k^2}[1 - (1 + kT)e^{-kT}]$. Since $(1 + kT)e^{-kT} \rightarrow 0$, total response is $A/k^2 = 24e$. At $T = 2$, $kT = 1$, so the fraction is $1 - 2/e$.

Marking: 1 mark for the finite integral; 1 for the improper limit; 1 for the total; 1 for the fraction.

(d) [2] The peak time $1/k$ and peak value $A/(ke)$ each halve. The total A/k^2 is quartered.

Marking: 1 mark for both peak changes; 1 for the total change.

Equivalent sound methods earn the same credit.