

RIGOROUS CALCULUS PRACTICE

European approaches

Calculus challenge mock

Selected A-level and French-style themes: models, sequences and proof

60 minutes | 3 structured questions | 30 marks

A graphing calculator may be used for numerical work. Exact answers and proofs are still required where requested.

This is a calculus-focused enrichment paper inspired by selected exam approaches. It is not a full national examination or a claim of syllabus equivalence. Europe has many different examination systems.

Before you begin

Write your name and date below. Keep the separate solutions file closed until you finish. Show equations, reasoning, units and required justifications. Use radian mode. Unless an exact answer is requested, round final numerical answers to three decimal places and retain unrounded values during calculations.

Name: _____ Date: _____

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Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 1: Exponential growth with a carrying capacity

10 marks

A population is modeled by $P' = 0.2P(1 - P/800)$, with $P(0) = 100$. Time is measured in years. This is a continuous population model; intermediate values need not be integers.

(a) Find the equilibrium solutions. For the given initial value, state the sign of P' and the long-term population predicted by the model. [2]

(b) Use separation of variables to show that $P(t) = 800/(1 + 7e^{-0.2t})$. [3]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 1: Exponential growth with a carrying capacity

10 marks

continued

Use the definitions and information on the preceding page.

(c) Find when the growth rate is greatest and calculate that greatest rate. [3]

(d) Find the first time the population reaches 600. Explain one reason a real population might depart from the long-term prediction. [2]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 2: Newton iteration and proof of convergence

10 marks

Let $u_0 = 2$ and $u_{n+1} = \frac{1}{2}(u_n + 2/u_n)$ for $n \geq 0$. The sequence is intended to approximate $\sqrt{2}$.

(a) Derive this iteration by applying Newton's method to $f(x) = x^2 - 2$, and calculate u_1 and u_2 exactly.
[2]

(b) Prove by induction that $u_n > \sqrt{2}$ for all n . You may derive an identity for $u_{n+1} - \sqrt{2}$. [3]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 2: Newton iteration and proof of convergence

10 marks

continued

Use the definitions and information on the preceding page.

(c) Prove that the sequence decreases and converges to $\sqrt{2}$.

[3]

(d) Show that $0 < u_{n+1} - \sqrt{2} \leq (u_n - \sqrt{2})^2 / (2\sqrt{2})$. Explain what this says about the error near the limit.
[2]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 3: A response curve and its parameters

10 marks

A nonnegative response is modeled by $C(t) = Ate^{-kt}$ for $t \geq 0$, where $A, k > 0$. Time is in hours. Measurements show that the peak occurs at $t = 2$ and the peak value is 12.

(a) Find the peak time and peak value in terms of A and k , with a derivative sign argument. [2]

(b) Determine A and k from the measurements. [2]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 3: A response curve and its parameters

10 marks

continued

Use the definitions and information on the preceding page.

(c) Find the total response $\int_0^\infty C(t)dt$ and the fraction of this total that occurs in the first 2 hours. Give exact answers. [4]

(d) If A is held fixed and k doubles, explain how the peak time, peak value, and total response change. [2]

Show sufficient work to support your answers.