

RIGOROUS CALCULUS PRACTICE

Singapore-inspired

Worked solutions and marking guide

Modeling, optimization and parametric reasoning

60 minutes | 3 structured questions | 30 marks

A graphing calculator may be used for numerical work. Exact answers and proofs are still required where requested.

This is a calculus-focused enrichment paper inspired by selected exam approaches. It is not a full national examination or a claim of syllabus equivalence. Europe has many different examination systems.

How to mark fairly

Award credit for mathematically equivalent methods. The point allocations are Academy practice rubrics, not official exam-board rubrics. Give method credit for sound work even if a later arithmetic error occurs; avoid penalizing the same error twice. Do not award a reasoning point for an unsupported correct number.

Add the marks across the three questions for a score out of 30. Use the topic labels to plan revision. No national grade conversion is implied.

Original questions created for Chen Math Academy. No questions are reproduced from Barron's, The Princeton Review, or an exam board. Independent preparation material; not affiliated with or endorsed by those publishers or College Board. AP is a College Board trademark.

Edition: September 17, 2026. ChenMathAcademy.org

Structured solution 1

Design an open box

Optimization and sensitivity

10 marks

(a) [2] The dimensions are x , $30 - 2x$, and $20 - 2x$, all positive precisely when $0 < x < 10$. Multiplication gives $V = x(30 - 2x)(20 - 2x) = 4x^3 - 100x^2 + 600x$.

Marking: 1 mark for the domain; 1 for the model and expansion.

(b) [3] $V' = 12x^2 - 200x + 600 = 0$ has roots $(25 \pm 5\sqrt{7})/3$. Only $x_* = (25 - 5\sqrt{7})/3 \approx 3.924$ cm is in $(0, 10)$.

Marking: 1 mark for the derivative; 1 for both algebraic roots; 1 for selecting the physical root.

(c) [3] V' is positive on $(0, x_*)$ and negative on $(x_*, 10)$; the other root exceeds 10. Also the limiting endpoint volumes are zero. Hence x_* is the absolute maximizer, with $V(x_*) \approx 1056.306$ cm³.

Marking: 1 mark for derivative signs; 1 for global/end behavior; 1 for maximum volume.

(d) [2] $V(4) = 1056$ cm³. Percentage loss is $100[V(x_*) - 1056]/V(x_*) \approx 0.029\%$. Because $V'(x_*) = 0$, the first-order change vanishes; the leading small change is quadratic in the displacement.

Marking: 1 mark for the percentage; 1 for the stationary-point explanation.

Equivalent sound methods earn the same credit.

Structured solution 2

Cooling and model interpretation

Differential equations, averages, modeling

10 marks

(a) [3] Separation gives $T - 22 = Ce^{-kt}$. Initially $C = 68$, and $34 = 68e^{-10k}$ implies $k = \ln 2/10 \text{ min}^{-1}$. Thus $T(t) = 22 + 68 \cdot 2^{-t/10}$.

Marking: 1 mark for solving the equation; 1 for the initial condition; 1 for determining k .

(b) [2] $T(25) = 22 + 68 \cdot 2^{-5/2} = 22 + 17/\sqrt{2} \approx 34.021^\circ\text{C}$.

Marking: 1 mark for substitution; 1 for the temperature.

(c) [3] The mean is $\frac{1}{20} \int_0^{20} (22 + 68e^{-kt}) dt = 22 + \frac{68(1-e^{-20k})}{20k} = 22 + \frac{25.5}{\ln 2} \approx 58.789^\circ\text{C}$.

Marking: 1 mark for average-value setup; 1 for integration/exact value; 1 for the decimal.

(d) [2] $8 = 68 \cdot 2^{-t/10}$ gives $t = 10 \ln(8.5)/\ln 2 \approx 30.875$ minutes. For example, room temperature must remain 22°C and the cooling constant must remain applicable.

Marking: 1 mark for time; 1 for a relevant assumption.

Equivalent sound methods earn the same credit.

Structured solution 3

A parametric region

Parametric calculus and geometric integrals

10 marks

(a) [3] $x' = 2t$, $y' = 3t^2 - 1$. The horizontal tangent occurs at $t = 1/\sqrt{3}$, giving $(1/3, -2/(3\sqrt{3}))$. At $t = 0$, $x' = 0$ while $y' = -1 \neq 0$, so the tangent at $(0, 0)$ is vertical (understood as the one-sided endpoint tangent).

Marking: 1 mark for derivatives; 1 for the horizontal point; 1 for the vertical point with justification.

(b) [2] $y \leq 0$ and x increases for $0 \leq t \leq 1$. Area $= -\int y dx = \int_0^1 (t - t^3) 2t dt = 2/3 - 2/5 = 4/15$.

Marking: 1 mark for the parameter integral including dx/dt ; 1 for the value.

(c) [3] $V = \pi \int_0^1 (t^3 - t)^2 (2t) dt = 2\pi \int_0^1 (t^7 - 2t^5 + t^3) dt = 2\pi(1/8 - 1/3 + 1/4) = \pi/12$.

Marking: 1 mark for disks and squared radius; 1 for the dx/dt factor and expansion; 1 for the volume.

(d) [2] $dy/dx = (3t^2 - 1)/(2t)$, so $d^2y/dx^2 = (3t^2 + 1)/(4t^3) > 0$ for $t > 0$. The curve is concave up as a graph of y against x .

Marking: 1 mark for the second derivative; 1 for its sign and interpretation.

Equivalent sound methods earn the same credit.