

RIGOROUS CALCULUS PRACTICE

Singapore-inspired

Calculus challenge mock

Modeling, optimization and parametric reasoning

60 minutes | 3 structured questions | 30 marks

A graphing calculator may be used for numerical work. Exact answers and proofs are still required where requested.

This is a calculus-focused enrichment paper inspired by selected exam approaches. It is not a full national examination or a claim of syllabus equivalence. Europe has many different examination systems.

Before you begin

Write your name and date below. Keep the separate solutions file closed until you finish. Show equations, reasoning, units and required justifications. Use radian mode. Unless an exact answer is requested, round final numerical answers to three decimal places and retain unrounded values during calculations.

Name: _____ Date: _____

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Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 1: Design an open box

10 marks

A $30\text{ cm} \times 20\text{ cm}$ rectangle of card is used to make an open box. Squares of side $x\text{ cm}$ are removed from its four corners, and the sides are folded up. Ignore the card thickness.

- (a) Give the physical domain of x and show that the volume is $V(x) = 4x^3 - 100x^2 + 600x$. [2]

- (b) Find the value of x giving the greatest volume. Give an exact value and a decimal approximation. [3]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 1: Design an open box

10 marks

continued

Use the definitions and information on the preceding page.

(c) Prove that the stationary point you selected is the absolute maximum on the physical domain. Find the maximum volume. [3]

(d) A designer chooses $x = 4$ cm. Calculate the percentage loss in volume compared with the optimum, and explain why the loss is relatively small near a stationary maximum. [2]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 2: Cooling and model interpretation

10 marks

A drink cools in a room at a constant 22°C . Its temperature satisfies $T' = -k(T - 22)$, where $k > 0$ and t is in minutes. Initially $T(0) = 90$ and $T(10) = 56$.

(a) Solve the differential equation and determine k exactly. [3]

(b) Find the predicted temperature at $t = 25$. [2]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 2: Cooling and model interpretation

10 marks

continued

Use the definitions and information on the preceding page.

(c) Find the average temperature during the first 20 minutes. Give an exact expression and a decimal value. [3]

(d) Find when the model first predicts $T = 30$. State one physical assumption that matters when using this prediction. [2]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 3: A parametric region

10 marks

A curve is given by $x = t^2$ and $y = t^3 - t$ for $t \geq 0$. Let R be the finite region between the curve and the x -axis for $0 \leq t \leq 1$.

(a) Find the horizontal and vertical tangent points for $t \geq 0$. Use derivative conditions to justify them. [3]

(b) Find the exact area of R . [2]

Show sufficient work to support your answers.

Structured challenge

Calculator permitted | 60 minutes for all three questions

Question 3: A parametric region

10 marks

continued

Use the definitions and information on the preceding page.

(c) Find the exact volume obtained by rotating R about the x -axis. [3]

(d) For $t > 0$, show that $d^2y/dx^2 > 0$ and interpret the result geometrically. [2]

Show sufficient work to support your answers.